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Statistical characterization and stochastic finite-element simulation of PMMA

Marcel Berlinger | Stefan Kolling | Jens Schneider

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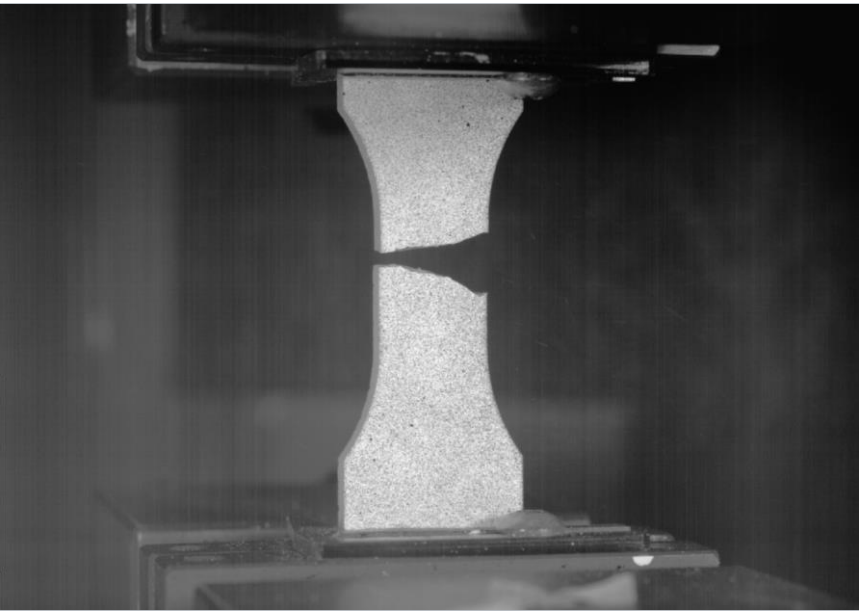

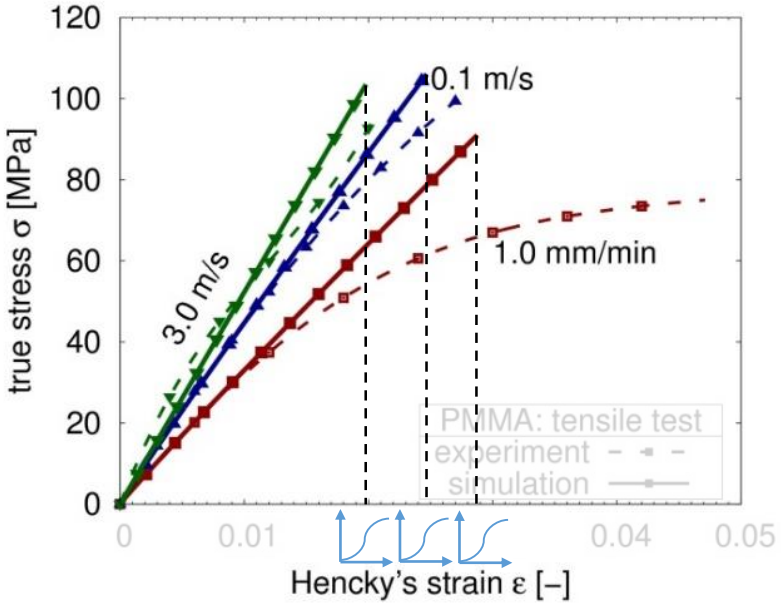

[1] Rühl, Andreas. On the time and temperature dependent behaviour of laminated amorphous polymers subjected to low-velocity impact. Vol.47, Springer, 2017

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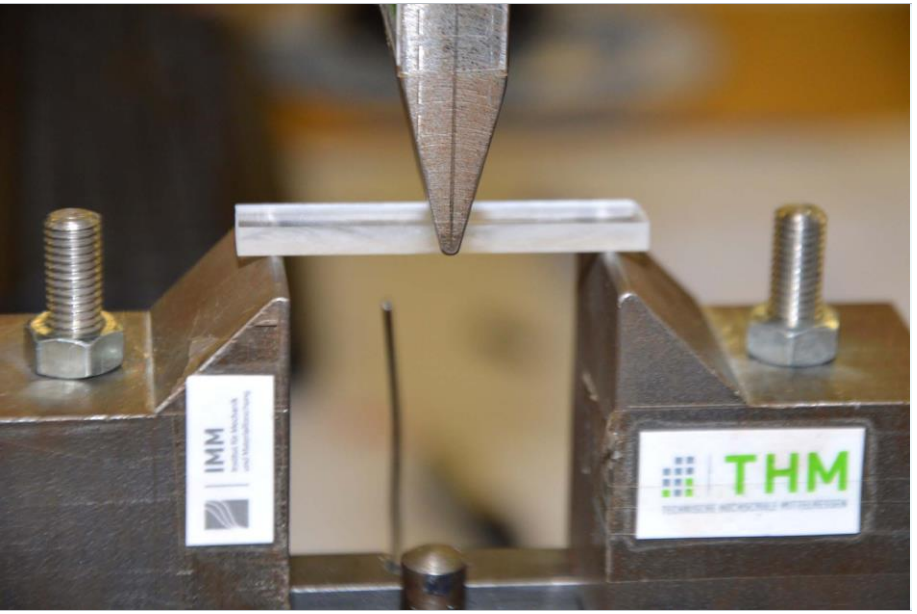

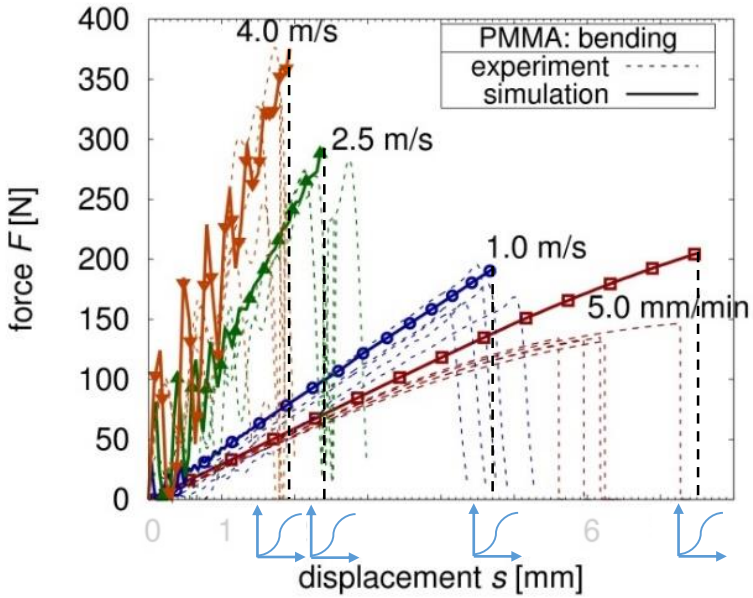

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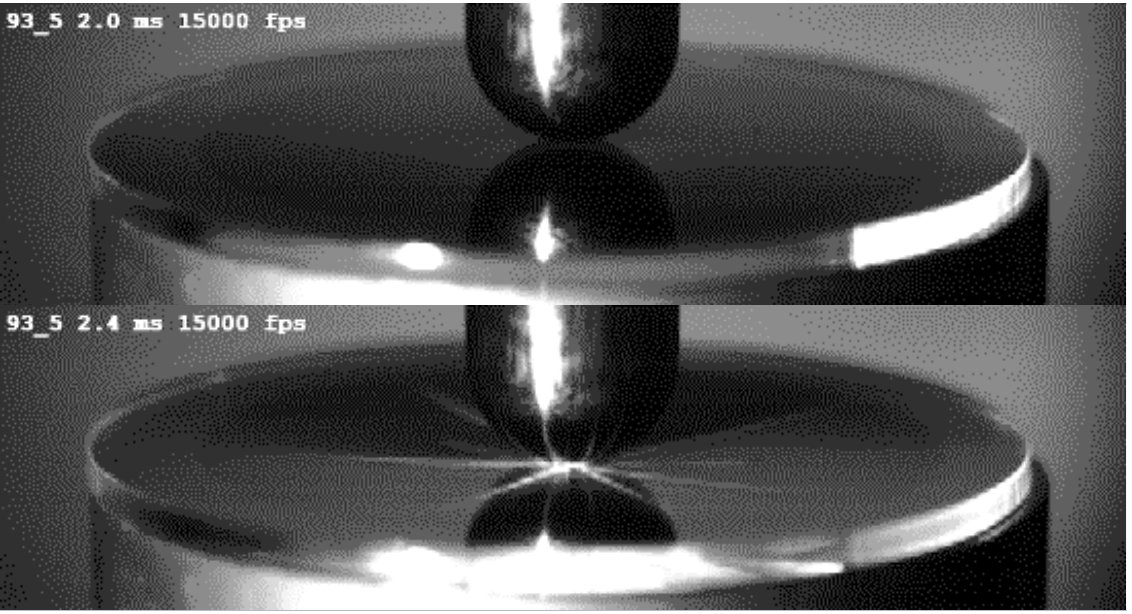

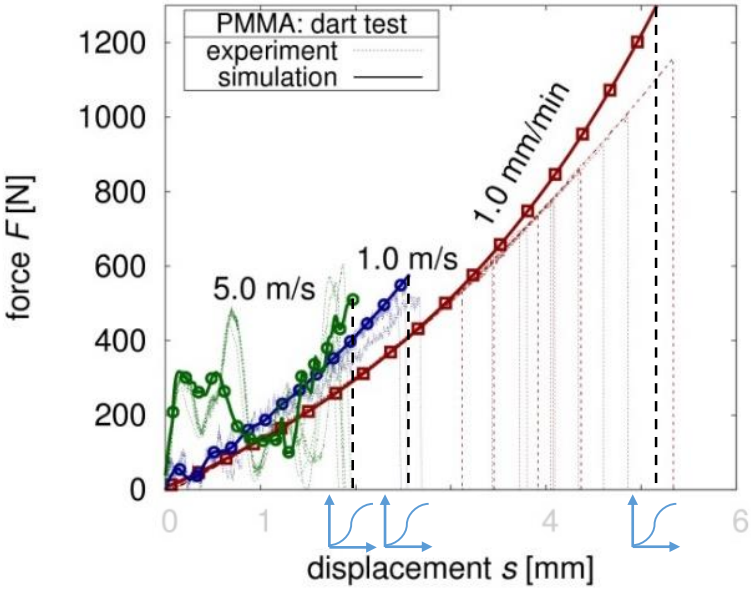

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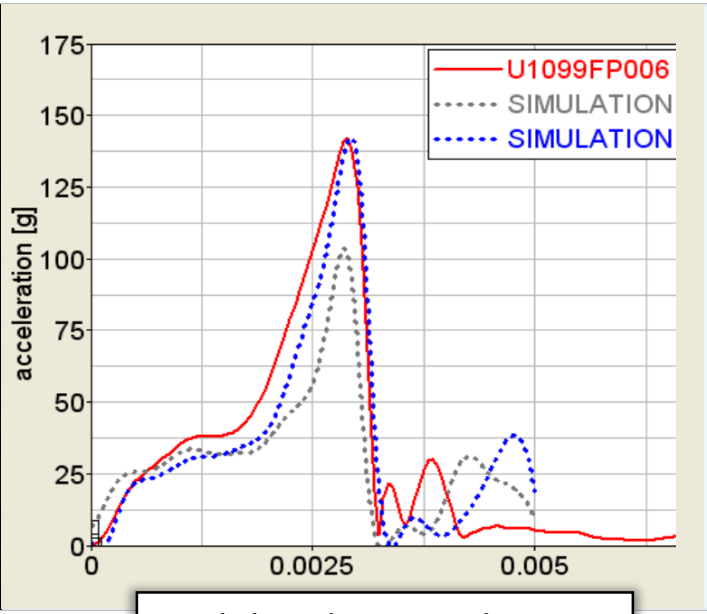

Validated to one distinct fracture strain.

[1] Rühl, Andreas. On the time and temperature dependent behaviour of laminated amorphous polymers subjected to low-velocity impact. Vol.47, Springer, 2017

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Content

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Experimental Research

Rate | Temperature | Triaxiality | DIC Analyses

Statistical Modeling

Probability Distribution Fit | Quantile Interpolation | Parameter Interpolation | Goodness-of-Fit | Open Topic

Stochastic Simulation

Random Erosion | Head Injury Criterion | HIC Distribution

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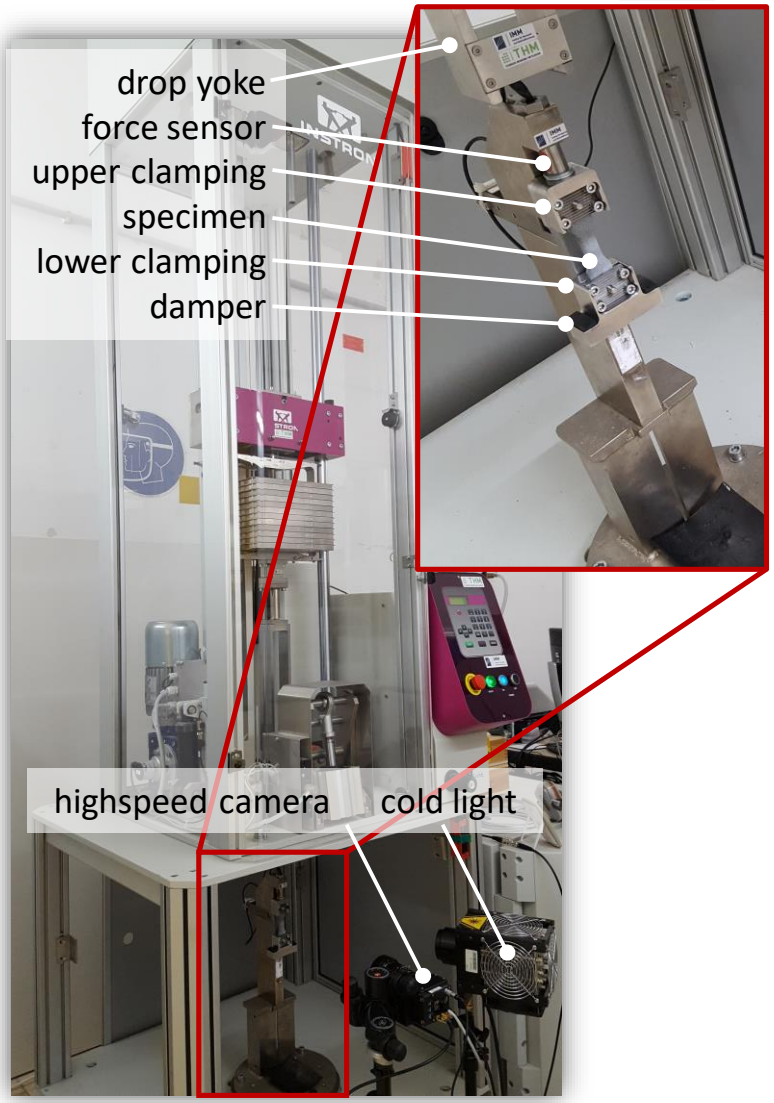
Experimental Research – Rate

- Tensile Tests at 7 different velocities
- Three different testing machines
  - Electro-mechanical system
  - Servo-hydraulic system
  - Drop-tower system
- In sum over 360 tensile tests

Ref.	Strain Rate [s^{-1}]	Testing Machine
$\dot{\varepsilon}_1$	4.6E+1	drop-tower system
$\dot{\varepsilon}_2$	3.8E+0	servo-hydraulical system
$\dot{\varepsilon}_3$	9.7E−1	servo-hydraulical system
$\dot{\varepsilon}_4$	1.6E−1	servo-hydraulical system
$\dot{\varepsilon}_5$	2.0E−2	servo-hydraulical system
$\dot{\varepsilon}_6$	2.0E−3	electro-mechanical system
$\dot{\varepsilon}_7$	1.6E−4	electro-mechanical system



Experimental Research – Temperature

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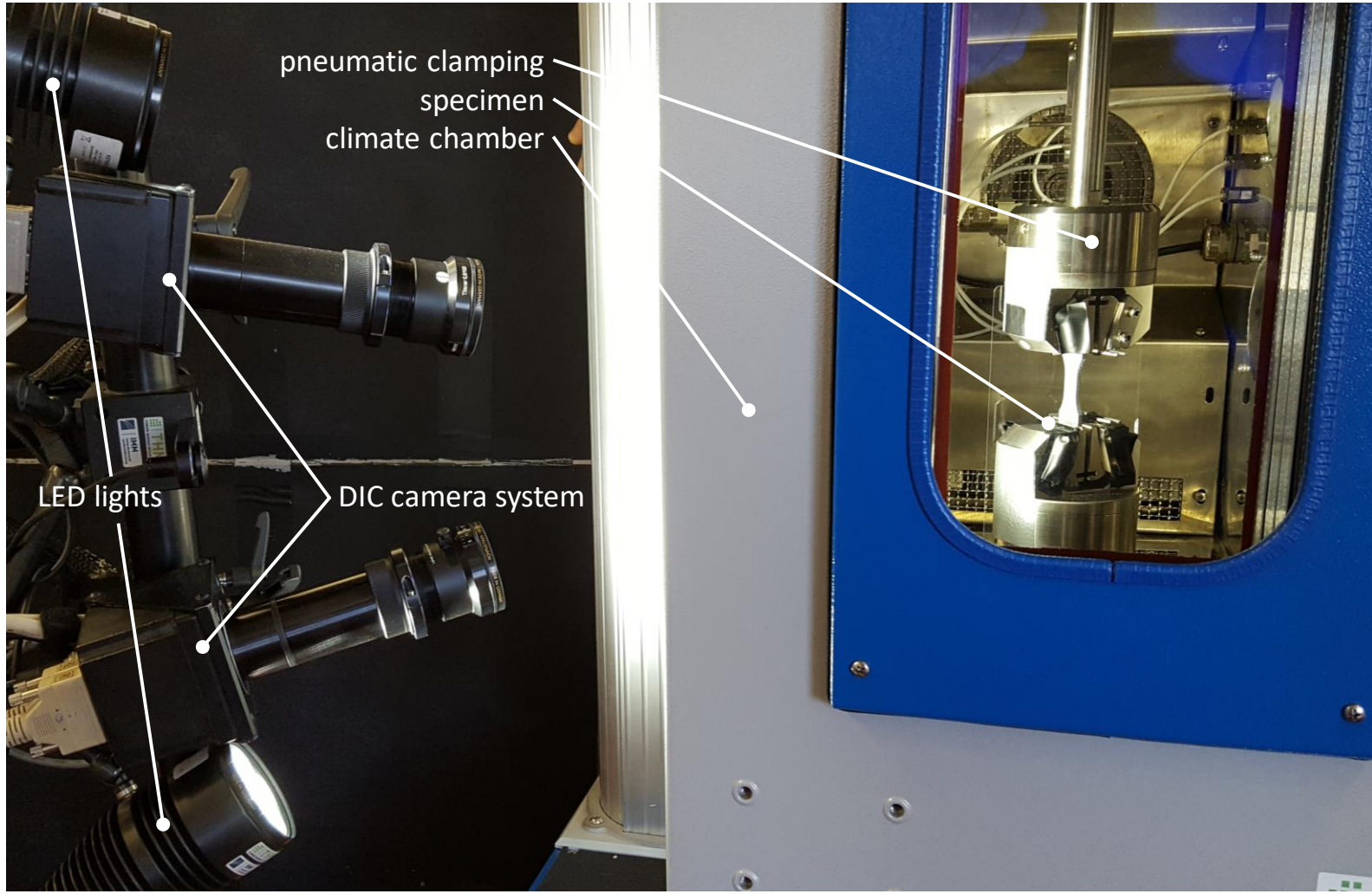
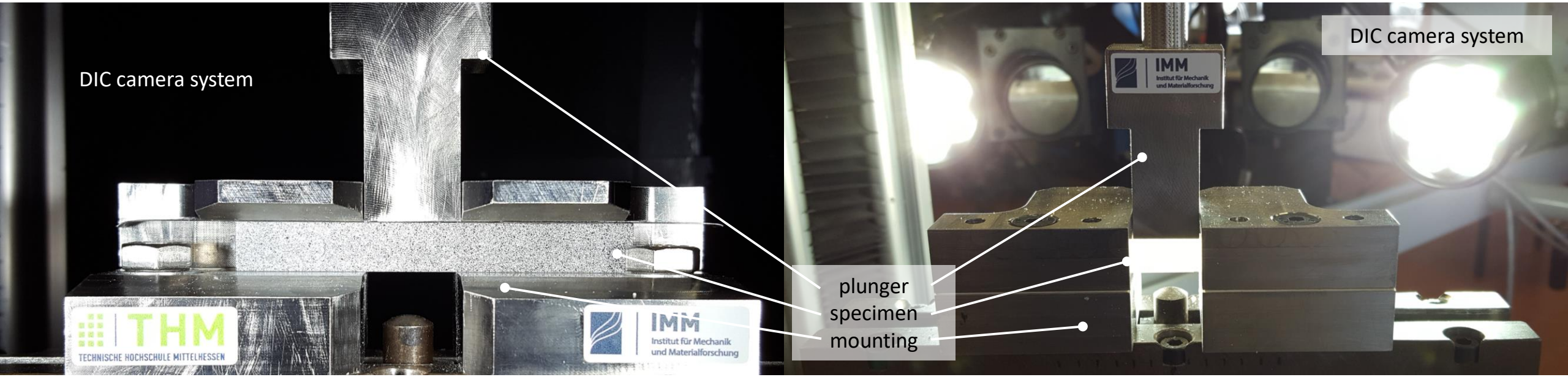
- Additional tensile tests in climate chamber
- Two common temperature limits for automotive applications
  - - 30 °C (H = 10 %)
  - + 85 °C (H = 40 %)
- 40 tests for each temperature



Experimental Research – Triaxiality

- Shear test on rectangular specimens (10mm x 80mm x 4mm)
- Constraint against rotation by tight fit in mounting
- Optical measurement of surface displacements via digital image correlation (DIC)



Experimental Research – Triaxiality

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- Puncture test on circular specimens (Ø90mm)
- Combination of spherical impactor (Ø20mm) and ring support (Ø42mm)
- Optical measurement of surface displacements via digital image correlation (DIC)

Session A

15:45	**Deformation analysis during puncture tests on plastics using 3D image correlation and numerical simulation** C. Jantos (THM)
16:10	**First results of full-field calibration applied to polymer materials** C. Ilg (Dynamore)
16:35	**IMPETUS™ - new efficient dynamic tensile test** S. Riemelmoser, C. Schober (4a engineering GmbH)

Experimental Research – DIC Analyses

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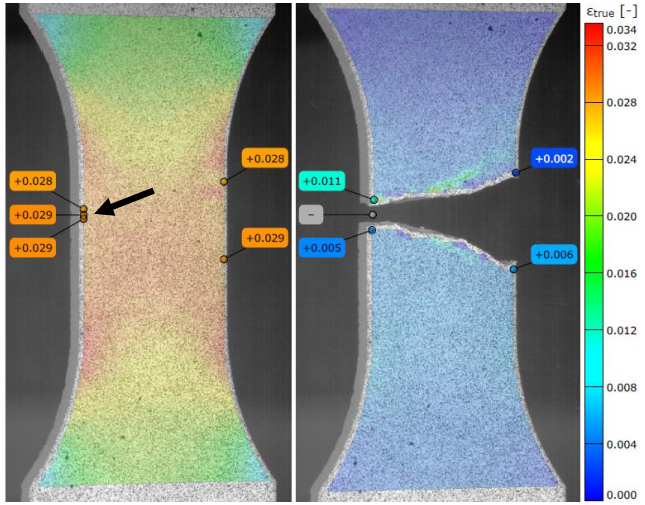
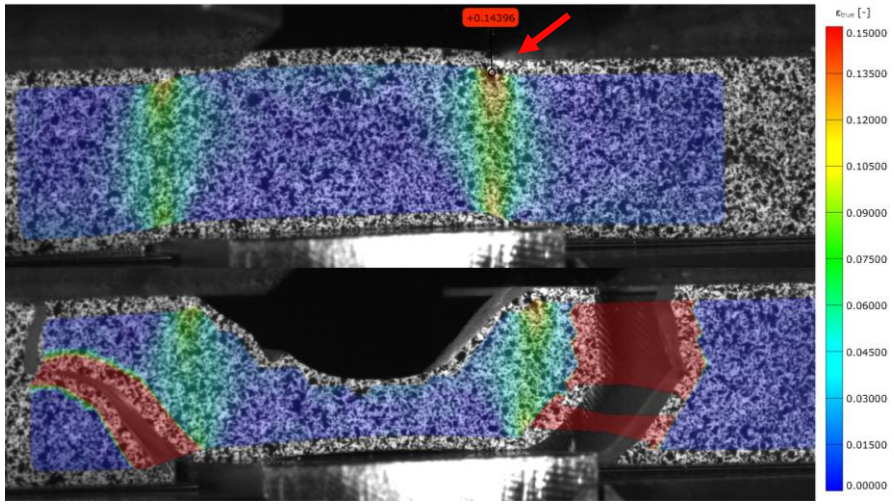
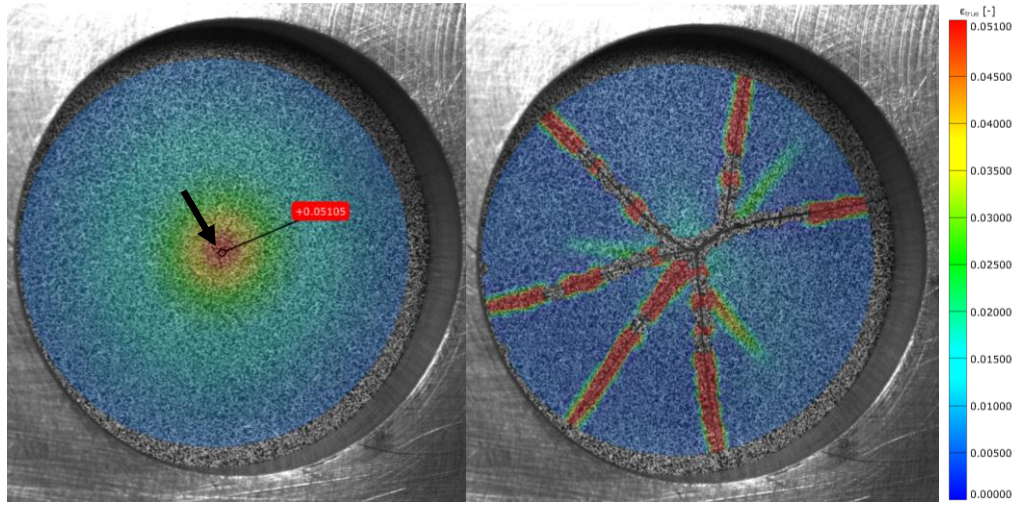
- GOM ARAMIS system for 3D digital image correlation (DIC) analyses
- Local fracture strains taken at the position of crack initiation
- Triaxiality level taken from FE-simulation (Open topic)







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Statistical Modeling – Probability Distribution Fit

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2-parameter Weibull distribution

$$P(\varepsilon) = 1 - \exp\left[-\left(\frac{\varepsilon}{\eta}\right)^{\beta}\right]$$

β – shape parameter
η – scale parameter

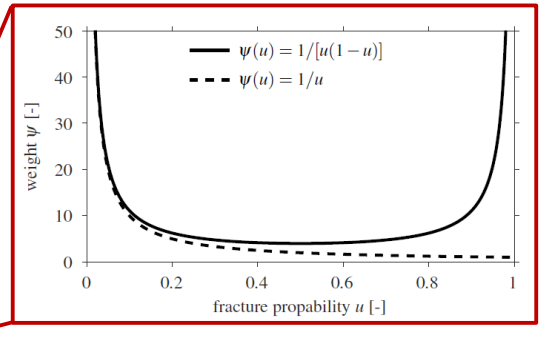
Probability estimator according to WEIBULL

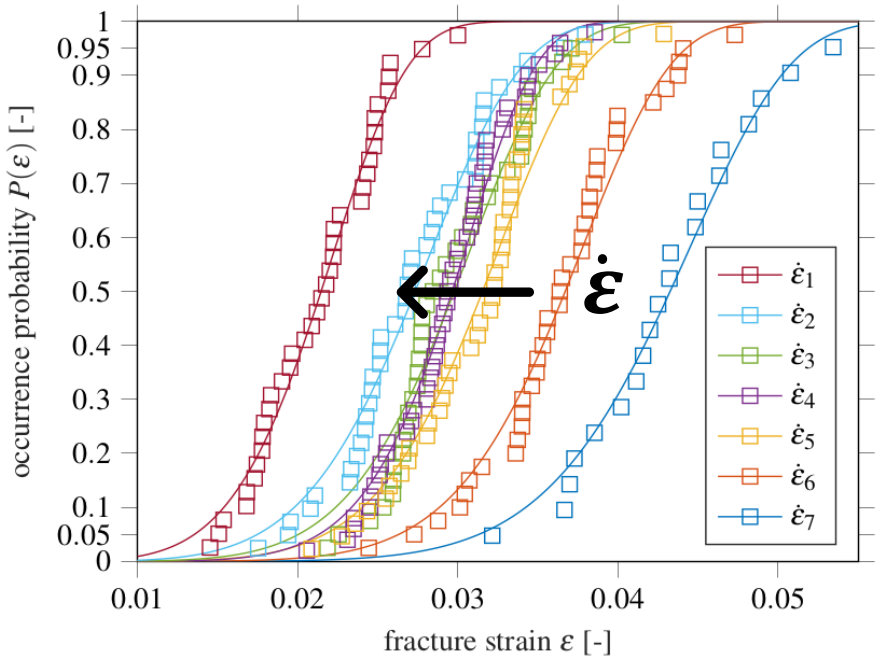
$$p_i = \frac{i}{n+1}$$

i – number in ascending order
n – sample size

Weighted residual sum of squared (WRSS)

$$WRSS = \sum_{i=1}^{n} [p_i - P(\varepsilon_i)]^2 \cdot \frac{1}{P(\varepsilon_i)[1 - P(\varepsilon_i)]}$$





Ref.	Strain Rate [s^{-1}]	Testing Machine
$\dot{\varepsilon}_1$	4.6E+1	drop-tower system
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Statistical Modeling – Quantile Interpolation

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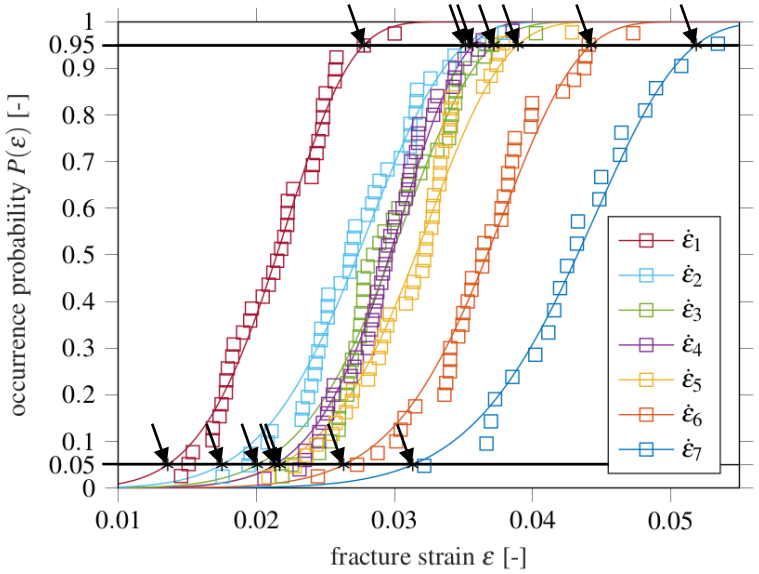
- Approach:

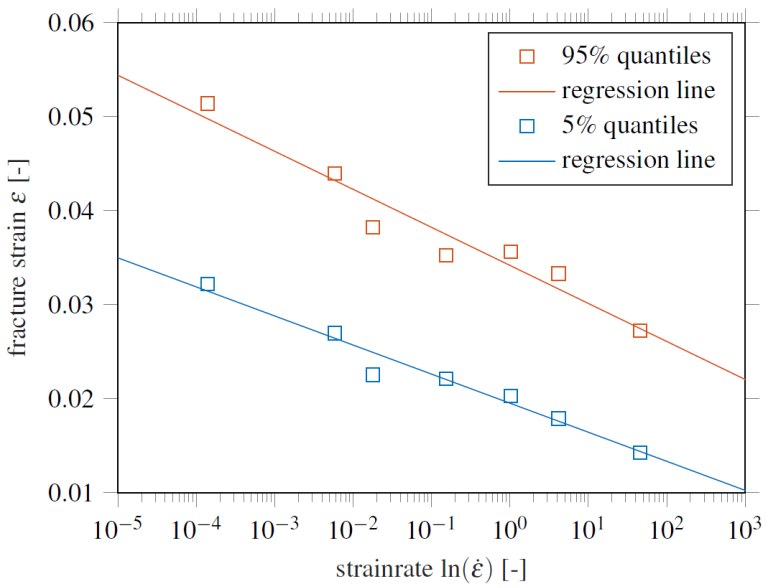
$$\varepsilon_{0.05} = f(\dot{\varepsilon})$$

$$\varepsilon_{0.95} = f(\dot{\varepsilon})$$

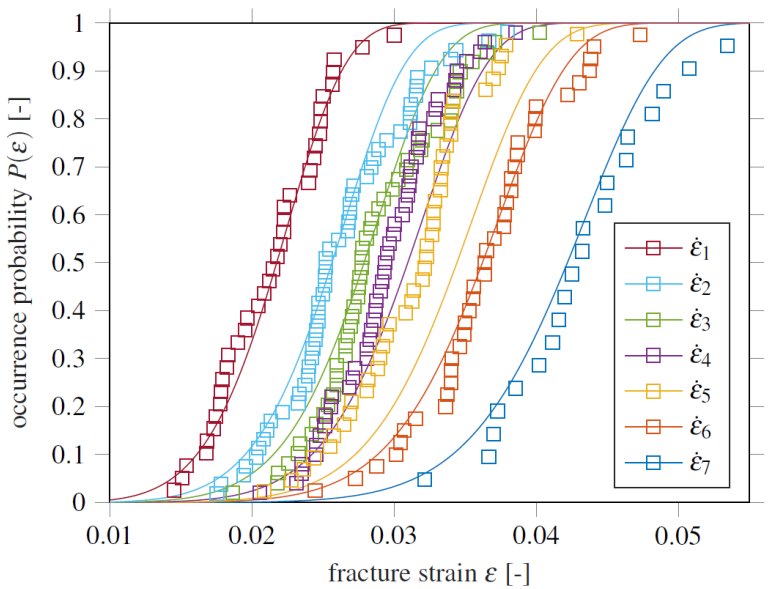


- Solving the system of equations:

$$\left| \begin{array}{l} 0.05 = 1 - \exp\left[-\left(\frac{\varepsilon_{0.05}}{\eta}\right)^{\beta}\right] \\ 0.95 = 1 - \exp\left[-\left(\frac{\varepsilon_{0.95}}{\eta}\right)^{\beta}\right] \end{array} \right|$$



- Model compared to measured fracture strains



Statistical Modeling – Parameter Interpolation

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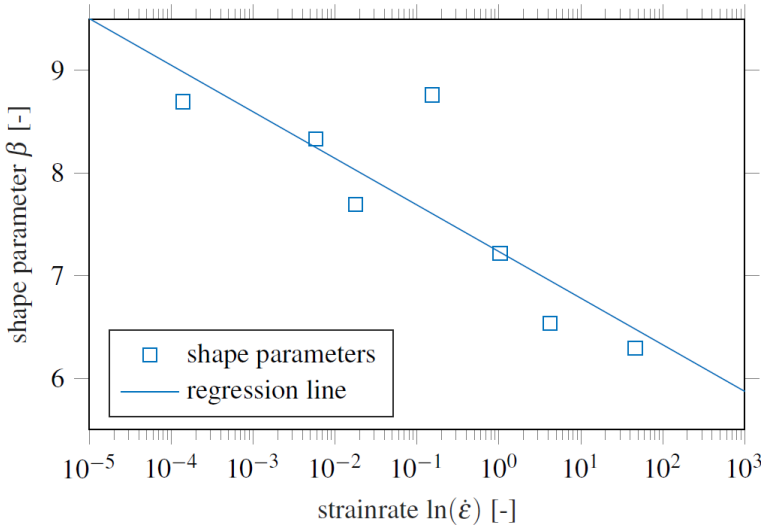
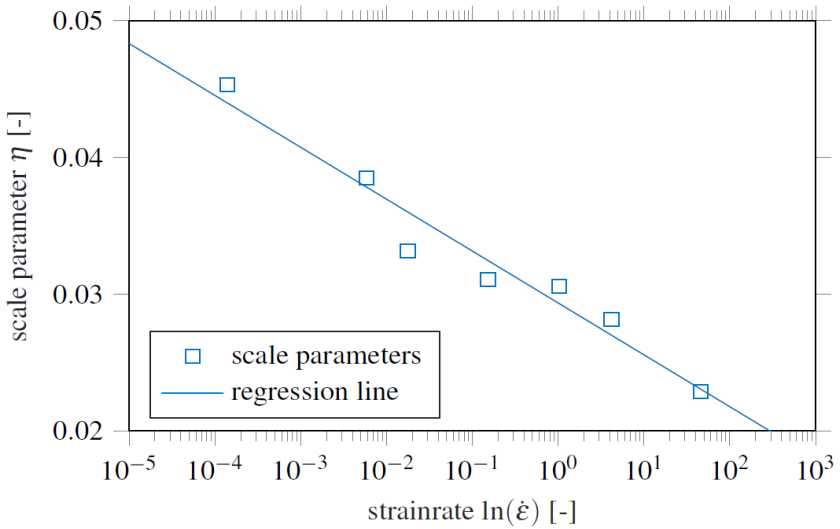
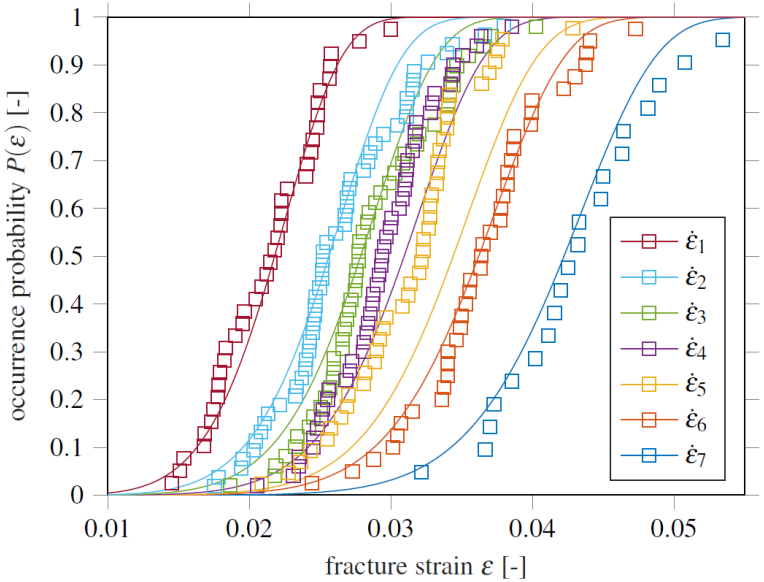
- Approach: $\beta = f(\dot{\varepsilon})$ & $\eta = f(\dot{\varepsilon})$





- Model compared to measured fracture strains



Statistical Model – Goodness-of-Fit

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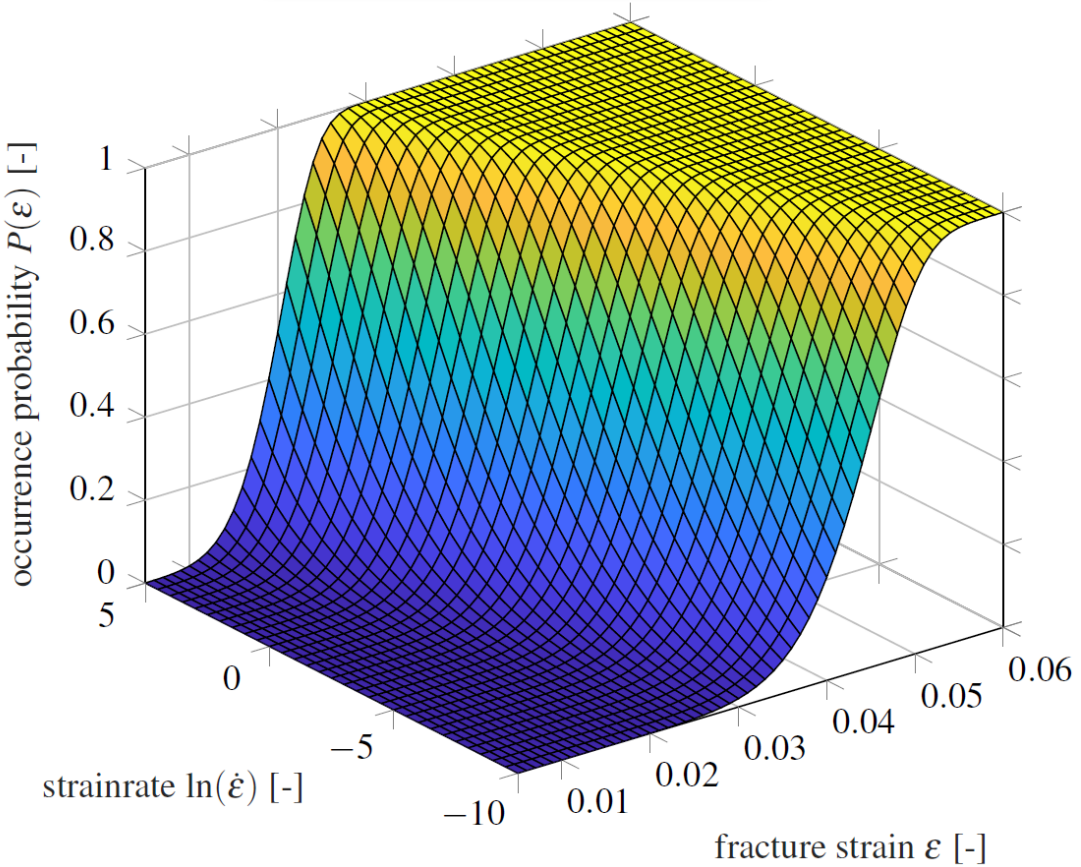
Coefficient of Determination

$$R^2 = 1 - \frac{\sum[p_i - P(\varepsilon_i)]^2}{\sum(p_i - \bar{p})^2}$$

Ref.	Original Fit	Quantile Modeled	Parameter Modeled
$\dot{\varepsilon}_1$	0.9819	0.9785	0.9746
$\dot{\varepsilon}_2$	0.9622	0.9575	0.9571
$\dot{\varepsilon}_3$	0.9461	0.9402	0.9391
$\dot{\varepsilon}_4$	0.9874	0.8554	0.8593
$\dot{\varepsilon}_5$	0.9784	0.5316	0.5377
$\dot{\varepsilon}_6$	0.9780	0.9766	0.9762
$\dot{\varepsilon}_7$	0.9846	0.9513	0.9535
Mean:	0.9741	0.8844	0.8853

Parameter Model



Statistical Modeling – Open Topic

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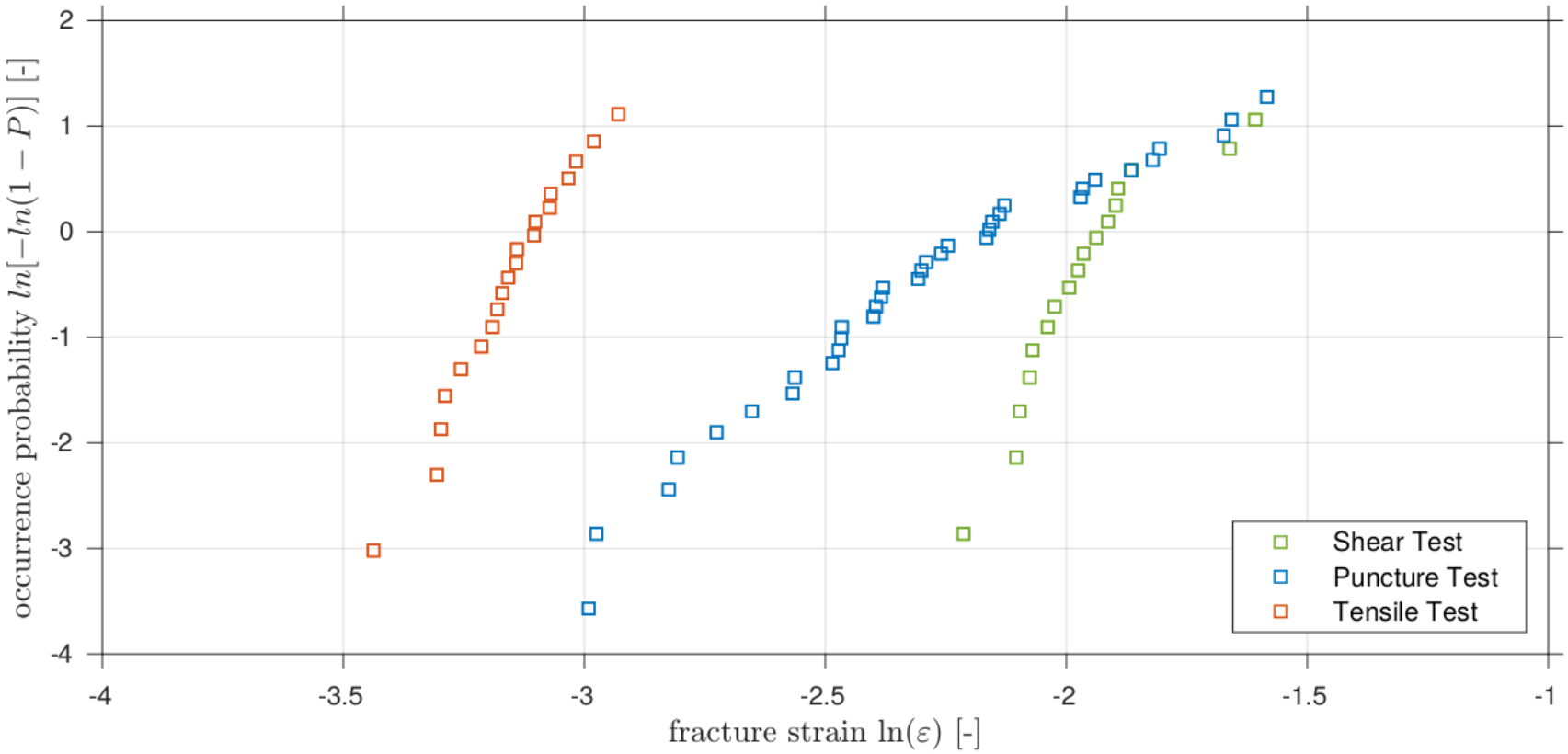
- Consideration of temperature and triaxiality level in statistical model
- Empirical data from shear, puncture and tensile tests:



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Stochastic Simulation – Random Erosion

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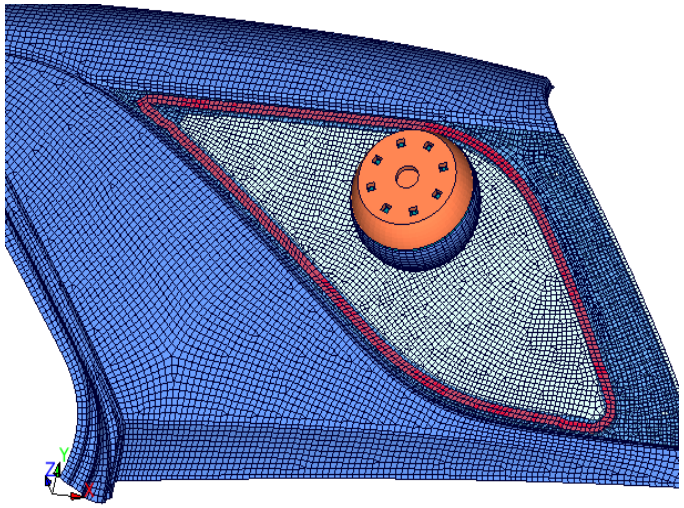
Statistical Model

$$P(\varepsilon, \dot{\varepsilon}) = 1 - \exp\left[-\left(\frac{\varepsilon}{\eta(\dot{\varepsilon})}\right)^{\beta(\dot{\varepsilon})}\right]$$

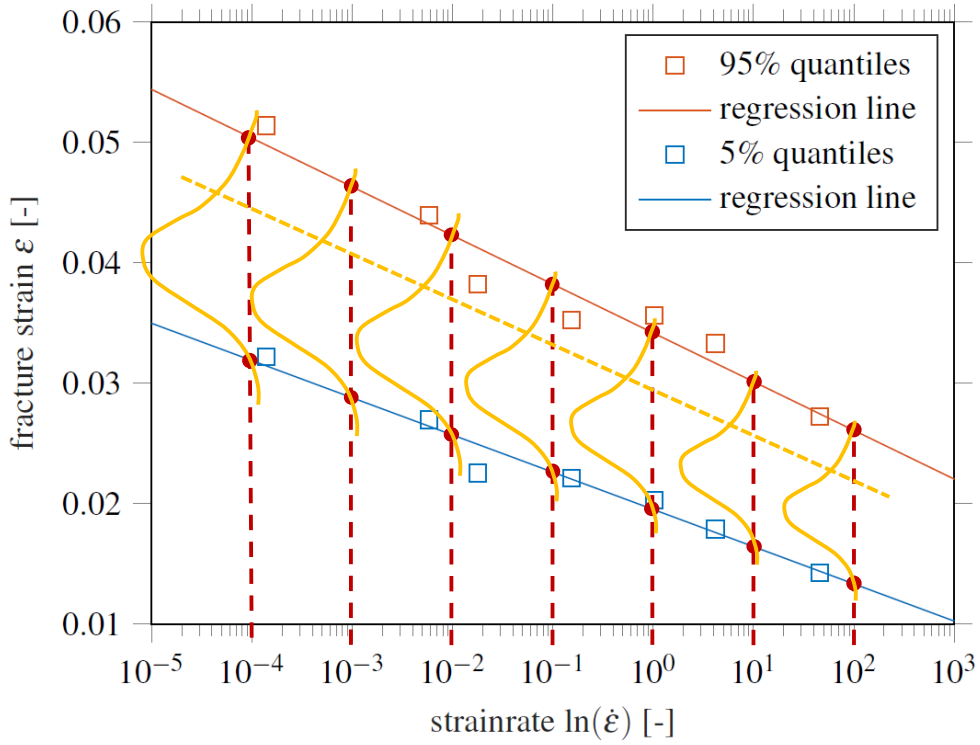
$$\varepsilon(P, \dot{\varepsilon}) = \eta(\dot{\varepsilon})\,[-\ln(1-P)]^{1/\beta(\dot{\varepsilon})}$$

representative rates

uniform random number (0,1)



Quantile Model



- Global fracture criterion
- “Resenable” amount of repetitions with random occurrence probability

Stochastic Simulation – Head Injury Criterion

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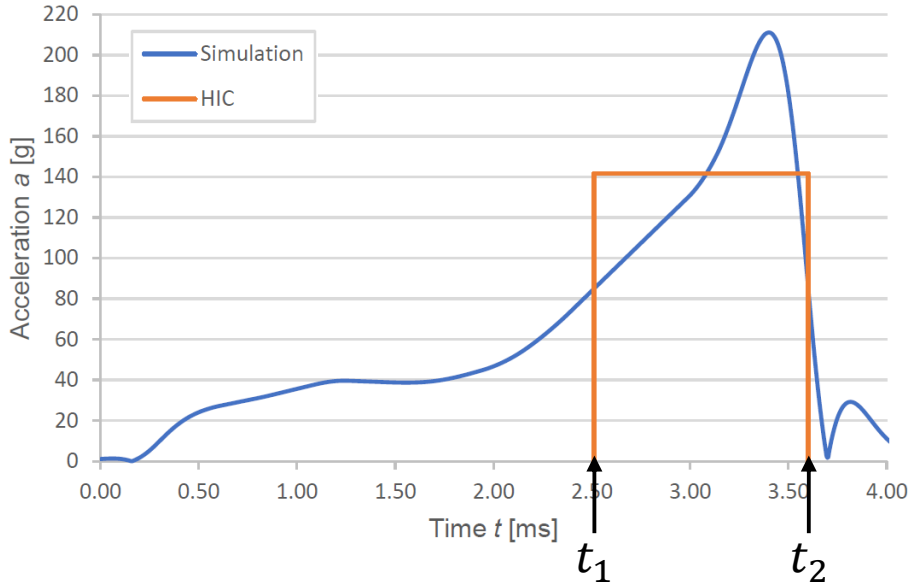
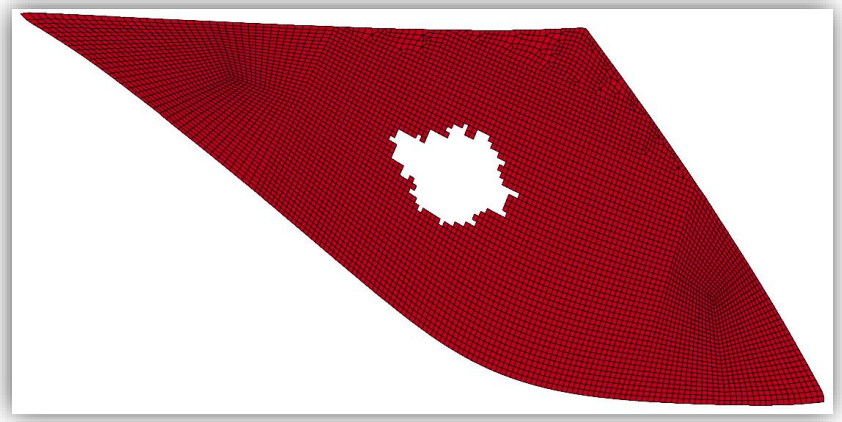
- Mesh size: 5mm edge length
- Resulting acceleration filtered by SAE J211 CFC-1000
- Head Injury Criterion (HIC) calculated by

$$HIC = \max\left\{\left[\frac{1}{t_2-t_1}\int_{t_1}^{t_2} a(t)\mathrm{d}t\right]^{2.5}(t_2-t_1)\right\}$$

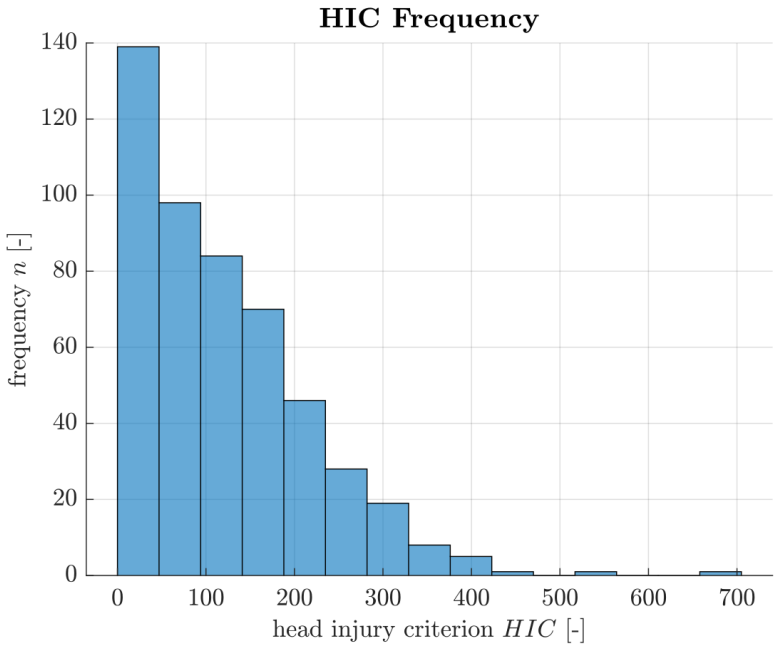
with respective choice of the time interval

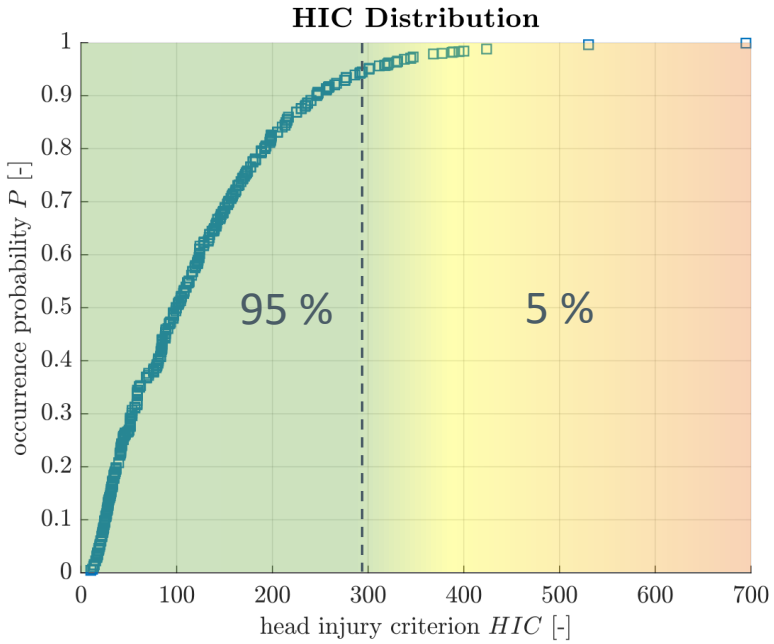




Stochastic Simulation – HIC Distribution

- 500 head impact simulations with random fracture criterion
- Head injury criteria result in secondary distribution





⚠ For the safety evaluation of structural parts a threshold quantile should be crucial

Summary & Outlook

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- Experimental research for empirical data
- Statistical interpolation model(s) for a strain-rate dependent fracture strain distribution
- FE-simulation with stochastic fracture criterion

- Consideration of temperature dependencies in the model
- Consideration of triaxiality dependencies in the model
- Stochastic FE-simulation with new model and re-assessment of the HIC distribution

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Institute for Mechanics and Materials

- Statistical characterization and modelling of polymer materials
- Stochastic FE-simulation

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