TECHNISCHE UNIVERSITÄT DARMSTADT

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Institute of Structural Mechanics and Design
Institut für Statik und Konstruktion

4a Technologietag 2018

Cavitation Failure in Rubber-Like Materials

A New Modeling Approach

Michael Drass M.Eng.

Chapter 1
Introduction

1. Introduction - Motivation
Adhesive Connections in Modern Glass Facades

Modern Structural Sealant Glazing Systems

- Adhesive connections in point supported façade systems
- Transparent structural silicone adhesive (TSSA) with strong adhesion performance under aesthetic and transparent look
- Hyperelastic material behavior
- Aspect ratio between height and diameter of silicone adhesive critical -> leading to high hydrostatic stresses within the bulk material

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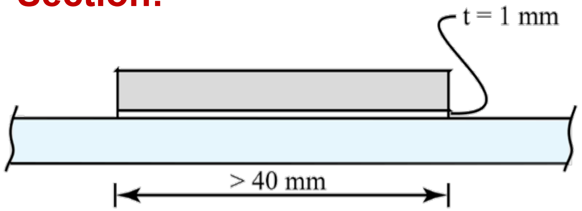

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1. Introduction - Motivation
Adhesive Connections in Modern Glass Facades

Modern Structural Sealant Glazing Systems

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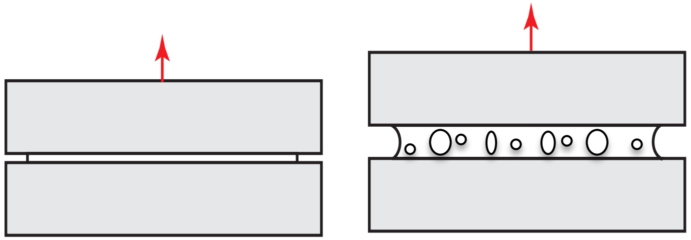
Cavitation & Stress Softening



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1. Introduction
Cavitation in Rubber-Like Materials

Cavitation Phenomenon

- Cavitation in rubber-like materials describes sudden void growth due to hydrostatic tension loading until material failure occurs
- Cavitation was analyzed experimentally, analytically and numerically
- Classical volumetric constitutive hyperelastic models cannot represent cavitation effect
- Numerical full-scale calculations accounting for cavitation are rare
- Many numerical approaches lack of a user-friendly environment in terms of finite element calculations and numerical robustness

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Chapter 2
Development of Volumetric Constitutive Model

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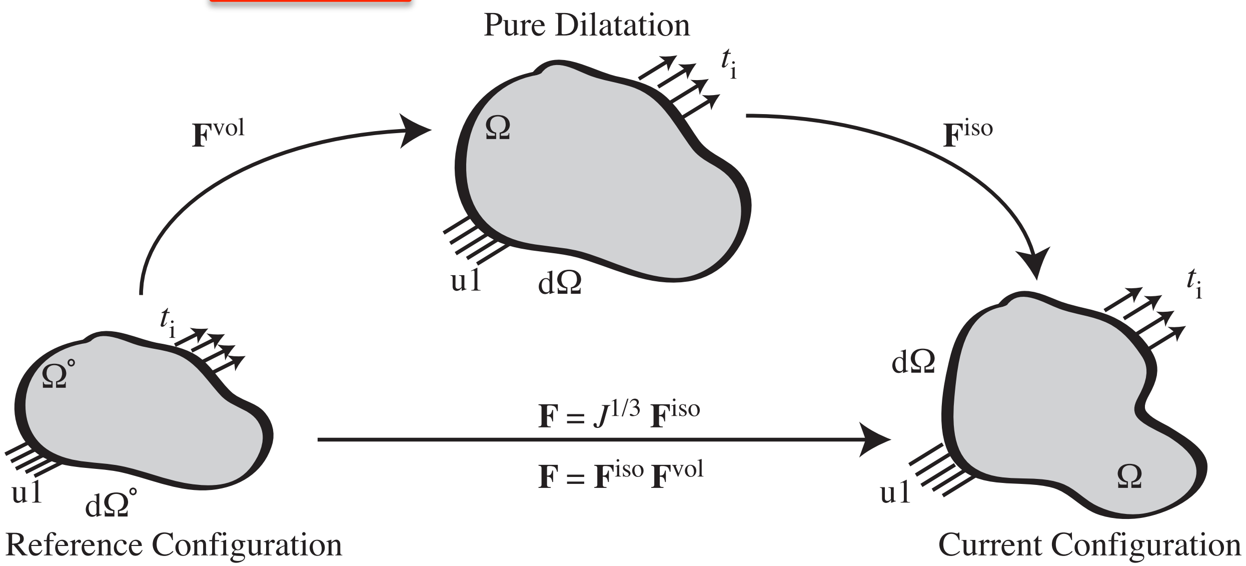
2. Development of Volumetric Constitutive Model
Basics on Continuum Mechanics

Compressible Hyperelastic Material Law

- Split of the Helmholtz free energy function into isochoric / volumetric components

$$\Psi\left(\mathbf{b}\right) = \Psi_{\text{iso}}\left(\bar{\mathbf{b}}\right) + \boxed{\Psi_{\text{vol}}\left(J\right)} \text{ with } \mathbf{F} = \mathbf{F}^{\text{iso}}\mathbf{F}^{\text{vol}}$$



2. Development of Volumetric Constitutive Model
Nano-Structure of Transparent Structural Silicone

Identification of a RVE (Periodic / Non-periodic Microstructure)

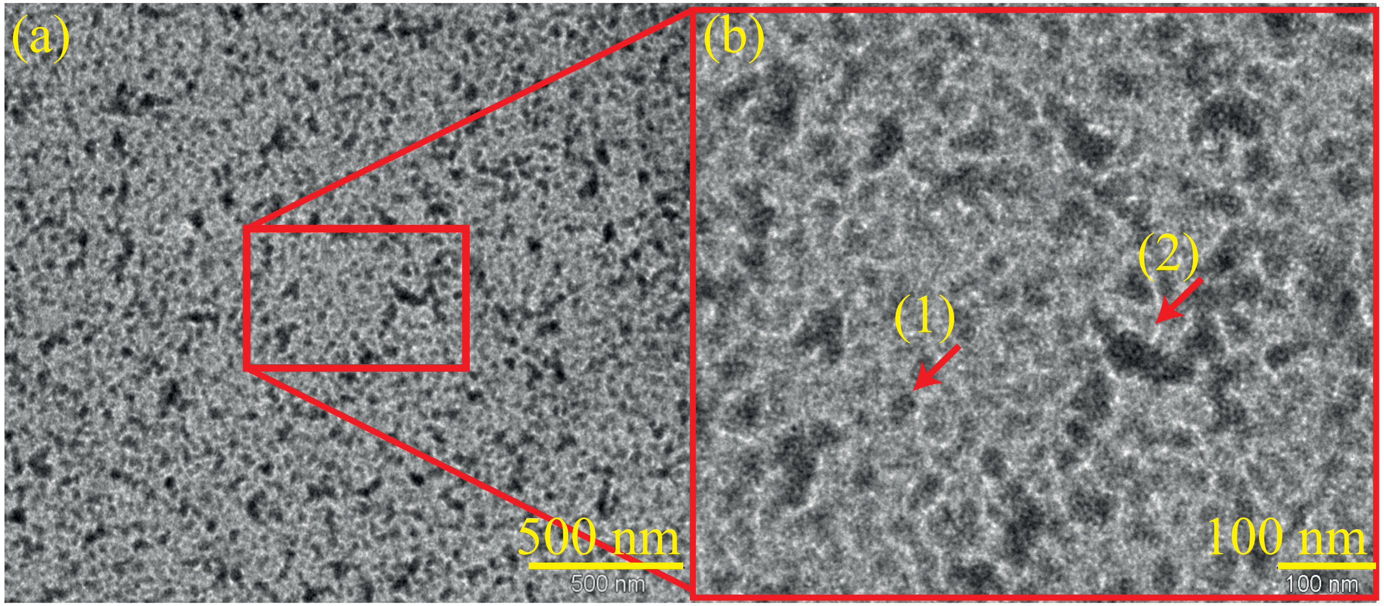
- TSSA bulk material consists of polydimethylsiloxane polymers
- High amount of nano-silica particles with d=1nm
- Multitude of nano-cavities developed during admixing

→ Built-up of numerical model accounting for nano-structure varying the fraction between bulk and cavity

2. Development of Volumetric Constitutive Model
Nano-Structure of Transparent Structural Silicone

Transmission Electron Microscopy (TEM) Images

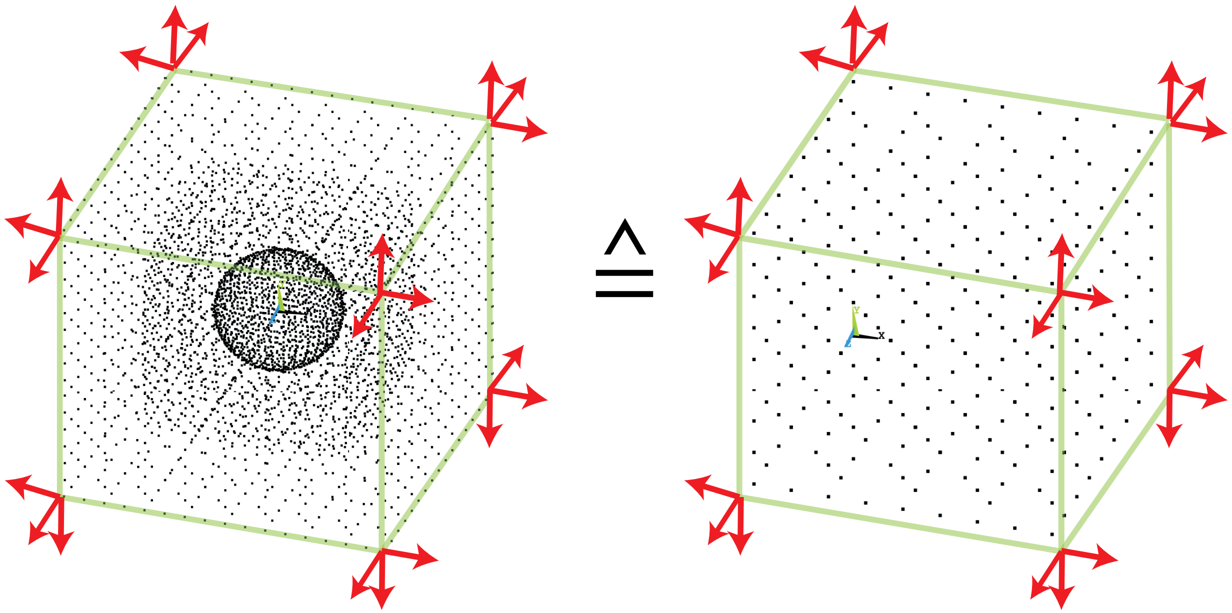
- Resolution ca. 1000 nm



- (1) void diameter ca. 30 nm; (2) void coalescence
- TSSA is a poro-hyperelastic material under finite void fraction

2. Development of Volumetric Constitutive Model Homogenization

Identification of a RVE (Periodic / Non-periodic microstructure)



- Dirichlet / Neumann boundary conditions applied

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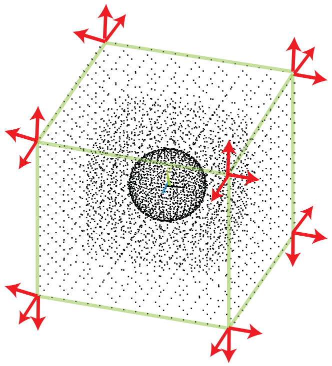
Rational Micro-Micro-Transition of Mechanical Fields

RVE



Periodic-BC

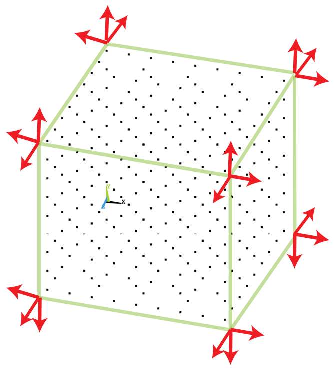
$$\bar{\sigma}_{ij} = \frac{1}{V^{RVE}} \int_{V^{RVE}} \sigma_{ij} \,\mathrm{d}V = \frac{1}{V^{EHM}} \int_{V^{EHM}} \sigma^*_{ij} \,\mathrm{d}V = \bar{\sigma}^*_{ij}$$

$$\bar{\varepsilon}_{ij} = \frac{1}{V^{RVE}} \int_{V^{RVE}} \varepsilon_{ij} \,\mathrm{d}V = \frac{1}{V^{EHM}} \int_{V^{EHM}} \varepsilon^*_{ij} \,\mathrm{d}V = \bar{\varepsilon}^*_{ij}$$

$$\boxed{\bar{\Psi} = \frac{1}{V^{RVE}} \int_{V^{RVE}} \Psi \,\mathrm{d}V = \frac{1}{V^{EHM}} \int_{V^{EHM}} \Psi^* \,\mathrm{d}V = \bar{\Psi}^*} \; *$$

$$\bar{F}_{ij} = \frac{1}{V^{RVE}} \int_{V^{RVE}} F_{ij} \,\mathrm{d}V = \frac{1}{V^{EHM}} \int_{V^{EHM}} F^*_{ij} \,\mathrm{d}V = \bar{F}^*_{ij}$$

EHM



Periodic-BC

* Suitable for large deformation

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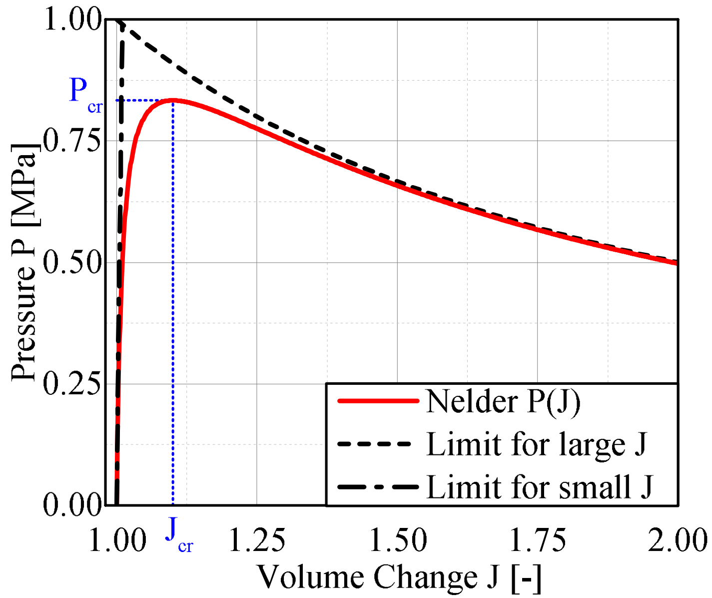
2. Development of Volumetric Constitutive Model Homogenization

Theory of Micromechanics and Homogenization

- Identification of a RVE (Periodic / Non-periodic microstructure)
- Rational micro-macro-transition of mechanical fields
- Determination of effective material properties

$$\frac{x}{y} = \beta_0 + \beta_1 x + \beta_2 x^2$$

$$\Rightarrow \qquad \bar{\Psi}^{*}_{\mathrm{vol,ND}} = \int_J \frac{\left(J-1\right)}{\sum_{i=0}^{m} \kappa_i \left(J-1\right)^i} dJ$$



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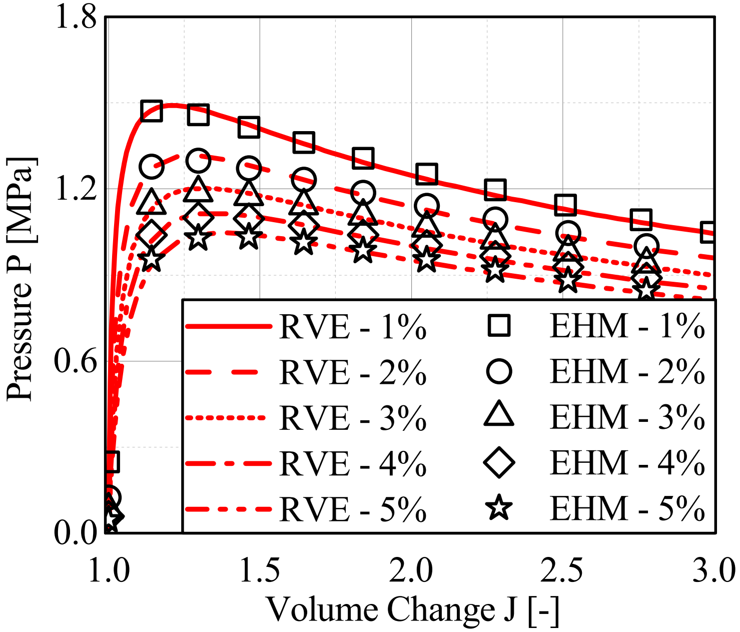
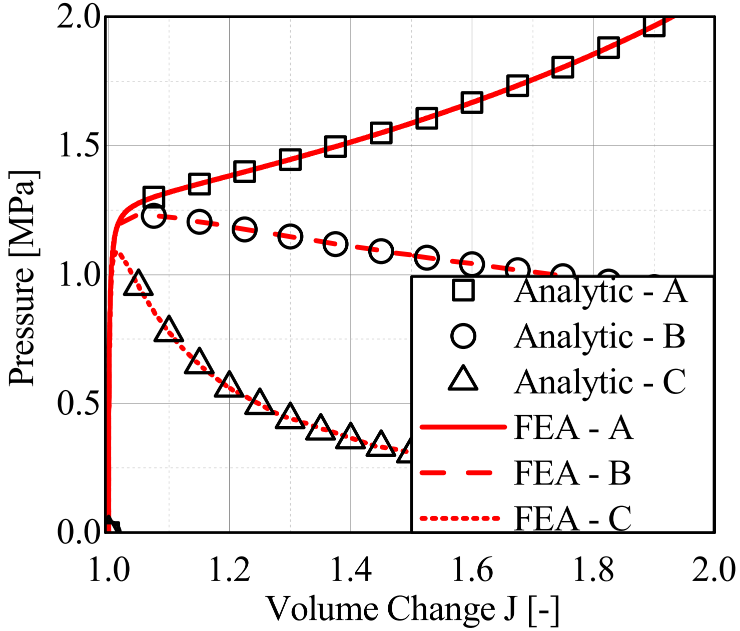
2. Development of Volumetric Constitutive Model
EHM vs. RVE

Results - Numerical Homogenization of RVE

- Excellent representation of the structural behavior of RVE with developed constitutive law





Chapter 3
Novel Modeling Approach

3. Helmholtz Free Energy Couple Term
Basic Idea

Isoperimetric, Volumetric Coupling Term Accounting for Cavitation

- To avoid volumetric softening during isochoric deformations, a couple term will be introduced with

$$\Psi = \Psi_{\text{iso}} + \Psi_{\text{vol,ND}} + \Psi_{\text{vol,couple}}$$

respectively

$$\Psi = \Psi_{\text{iso}} + \Psi_{\text{vol,ND}} + \Omega\left(\Psi_{\text{vol,classic}} - \Psi_{\text{vol,ND}}\right)$$

- In this context Ω is defined as isoperimetric, volumetric shape function with

$$\Omega\left(\Pi,\Theta\right) = \left(1-\Pi\right)\Theta \text{ with } \Omega \in \left[0,1\right]$$

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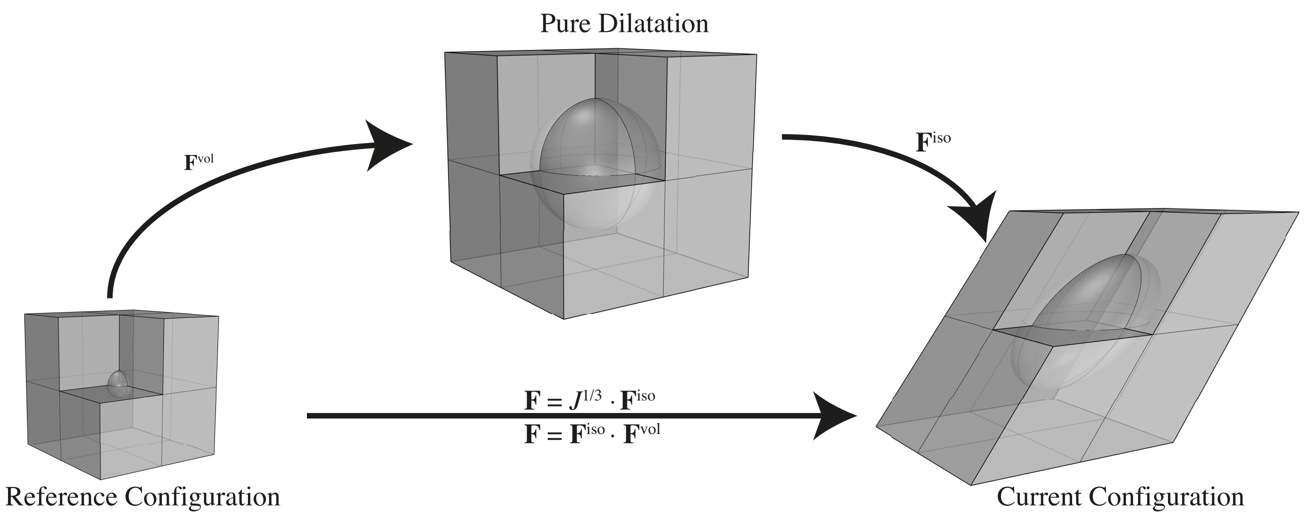
3. Helmholtz Free Energy Couple Term
Basic Idea

Isoperimetric, Volumetric Coupling Term Accounting for Cavitation

- The volumetric shape function Θ defines weather a void is growing in volume
- The isoperimetric inequality Π describes how the void is geometrically growing



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3. Helmholtz Free Energy Couple Term Isoperimetric Inequality

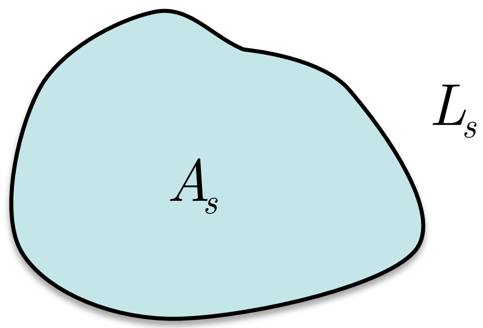
Definition of Isoperimetric Inequality

- The isoperimetric inequality is a fundamental problem in classical calculus of variations
- It is described by finding the geometric figure with maximal area at a given perimeter
- In 2D Euclidean space, it is defined by

$$t = \frac{4\pi A_s}{L_s} \leq 1 \text{ with } t \in \left[0,1\right]$$

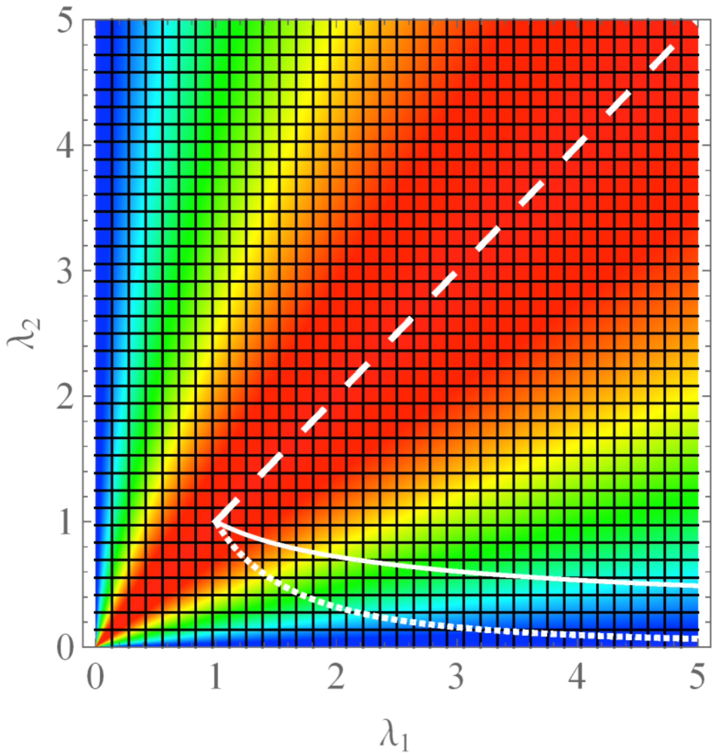
L_s = Length of a simple closed curve s

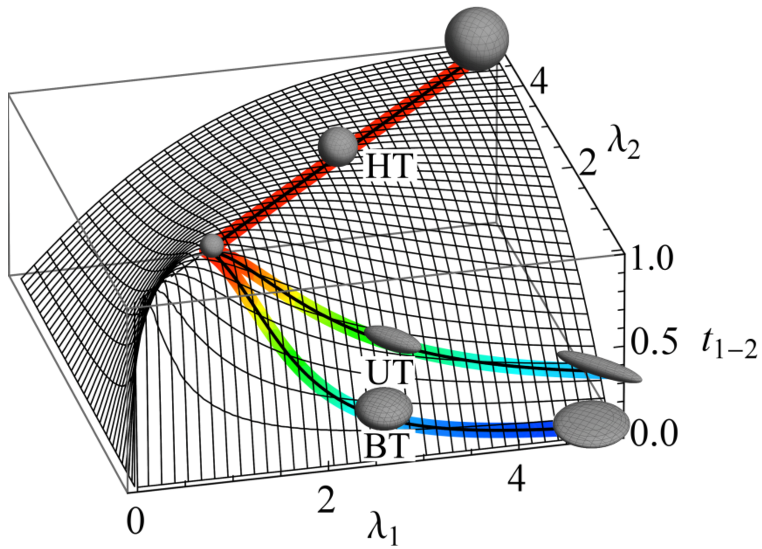
A_s = Area of the closed curve s



3. Helmholtz Free Energy Couple Term
Isoperimetric Inequality

Plot of Isoperimetric Inequality – n = 1





$$t = \left(\frac{4\pi A_s}{L_s} \right)^n \leq 1 \text{ with } t \in [0,1]$$

3. Helmholtz Free Energy Couple Term
Equivalent Volumetric Strain

Definition

- Introduction of equivalent volumetric strain (Hencky Strains)

$$\varepsilon_{\mathrm{eqn,vol}} = I_\varepsilon + II_\varepsilon + III_\varepsilon \qquad \text{with} \quad I_\varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

$$II_\varepsilon = \varepsilon_1 \varepsilon_2 + \varepsilon_2 \varepsilon_3 + \varepsilon_1 \varepsilon_3$$

$$III_\varepsilon = \varepsilon_1 \varepsilon_2 \varepsilon_3$$

- Normalization of equivalent volumetric strain to obtain single scalar, which differentiates between isochoric and volumetric deformations

$$\Theta = \frac{\dfrac{2\varepsilon_{\mathrm{eqn,vol}}}{\left|\varepsilon_{\mathrm{eqn,vol}}\right|} + 2}{4} \text{ with } \Theta \in \left[0,1\right]$$

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Chapter 4
Computational Validation

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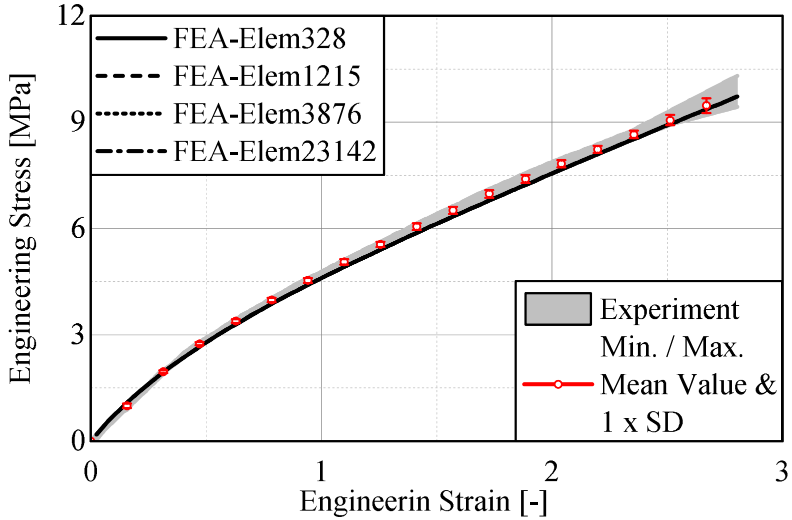
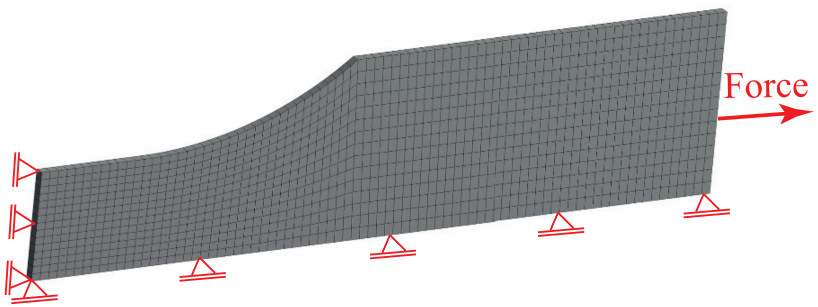
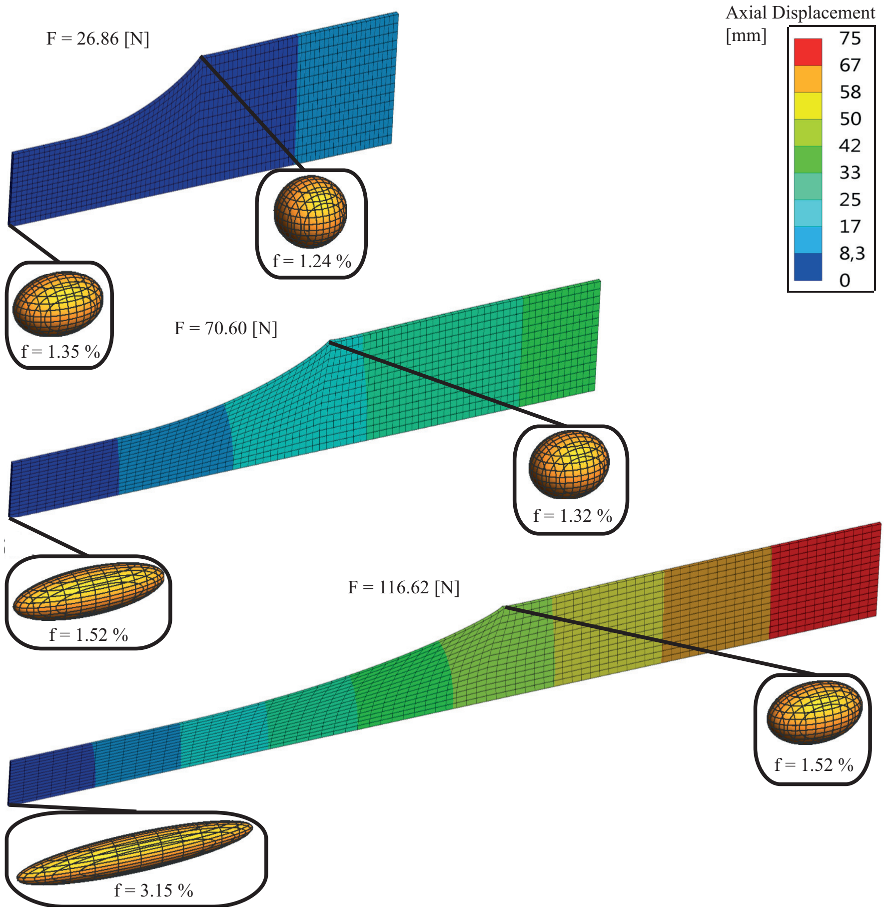
4. Computational Validation

Simulation of Tension Test







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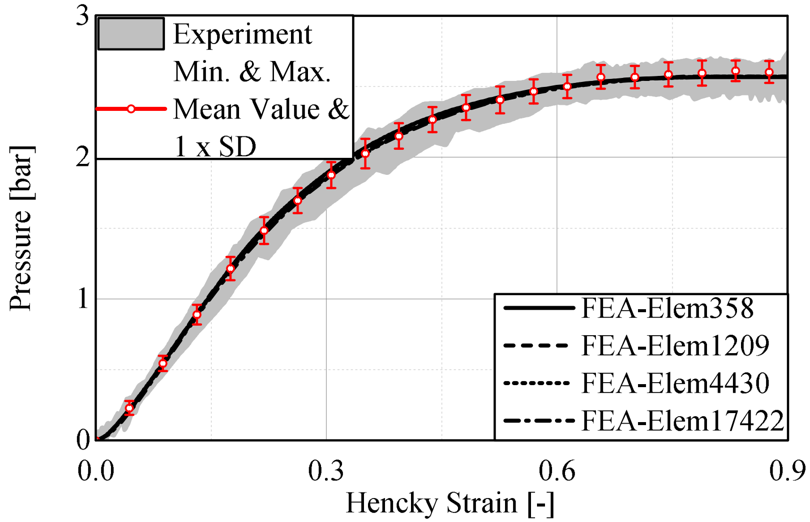
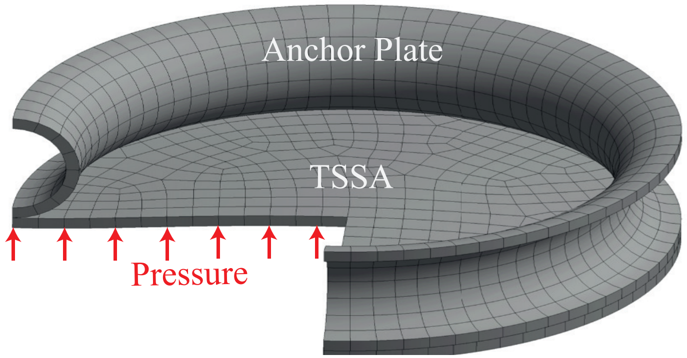
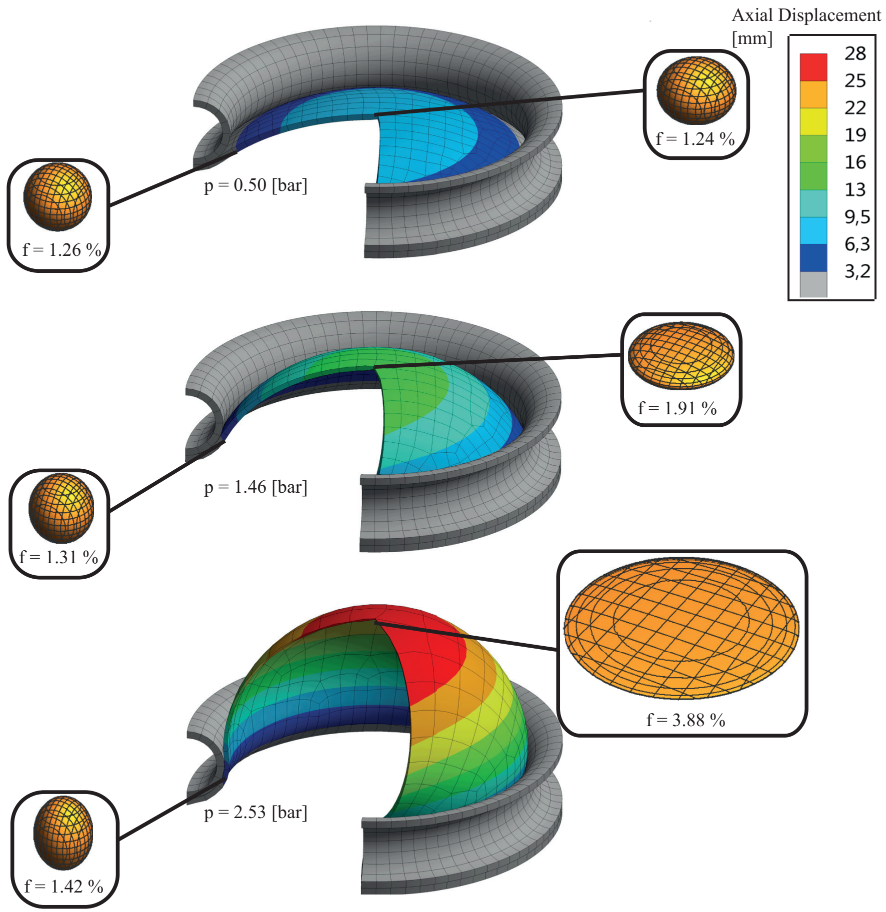
4. Computational Validation

Simulation of Bulge Test







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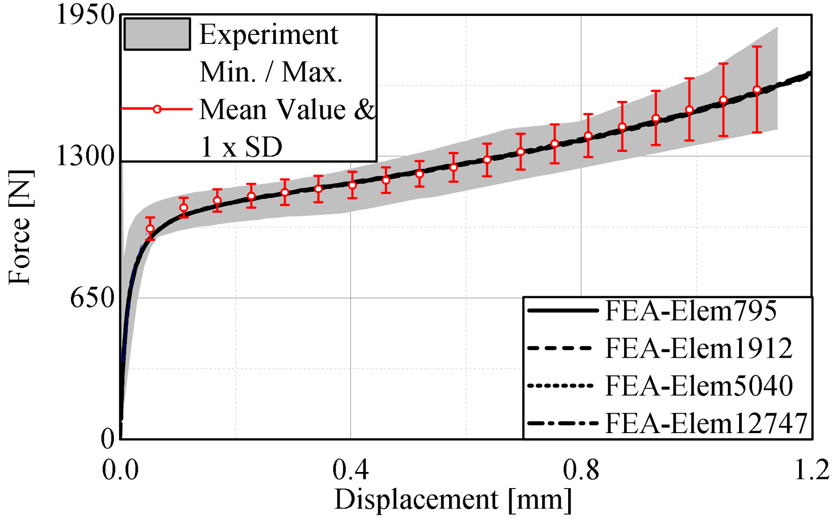
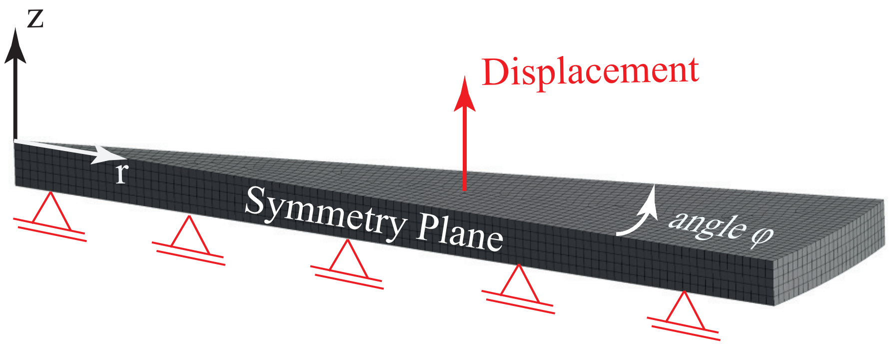
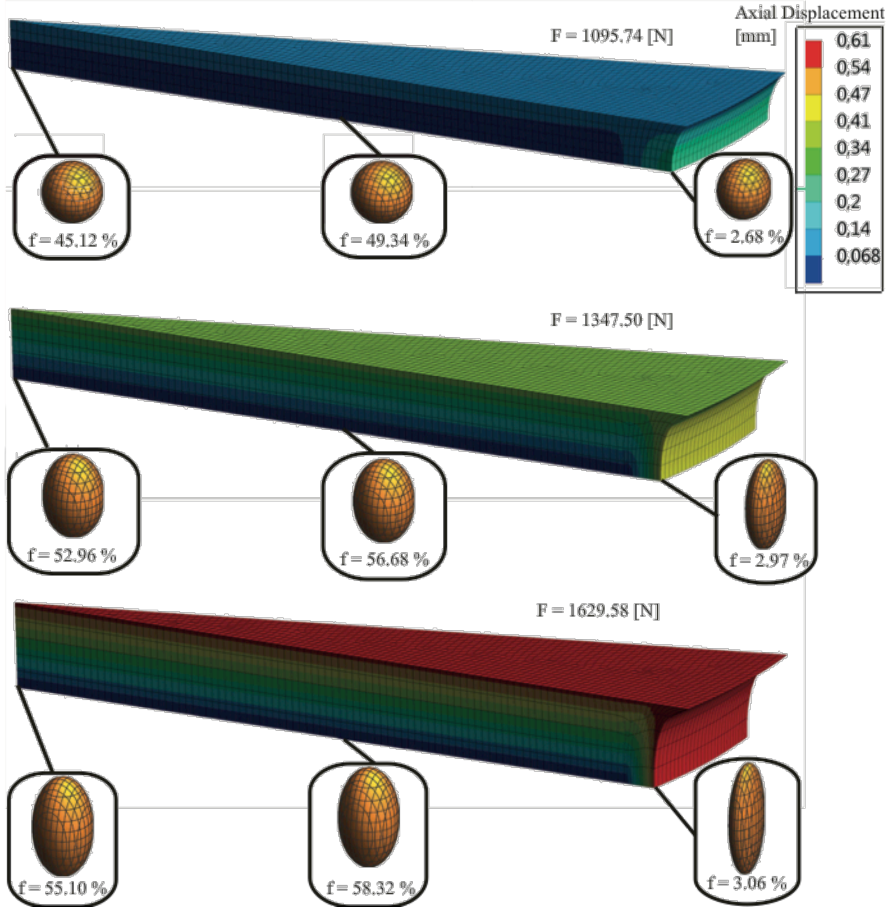
4. Computational Validation

Simulation of Pancake Tension Test







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Thank your for your attention!