Technologietag 2017: Leichtbau und Composites

Interface stresses in bilayered composites - analytical and numerical considerations

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Outline

- Motivation
- Preliminary Considerations
- Analytical Approach
- Results of the Analytical Approach
- Numerical Approach
- Conclusions

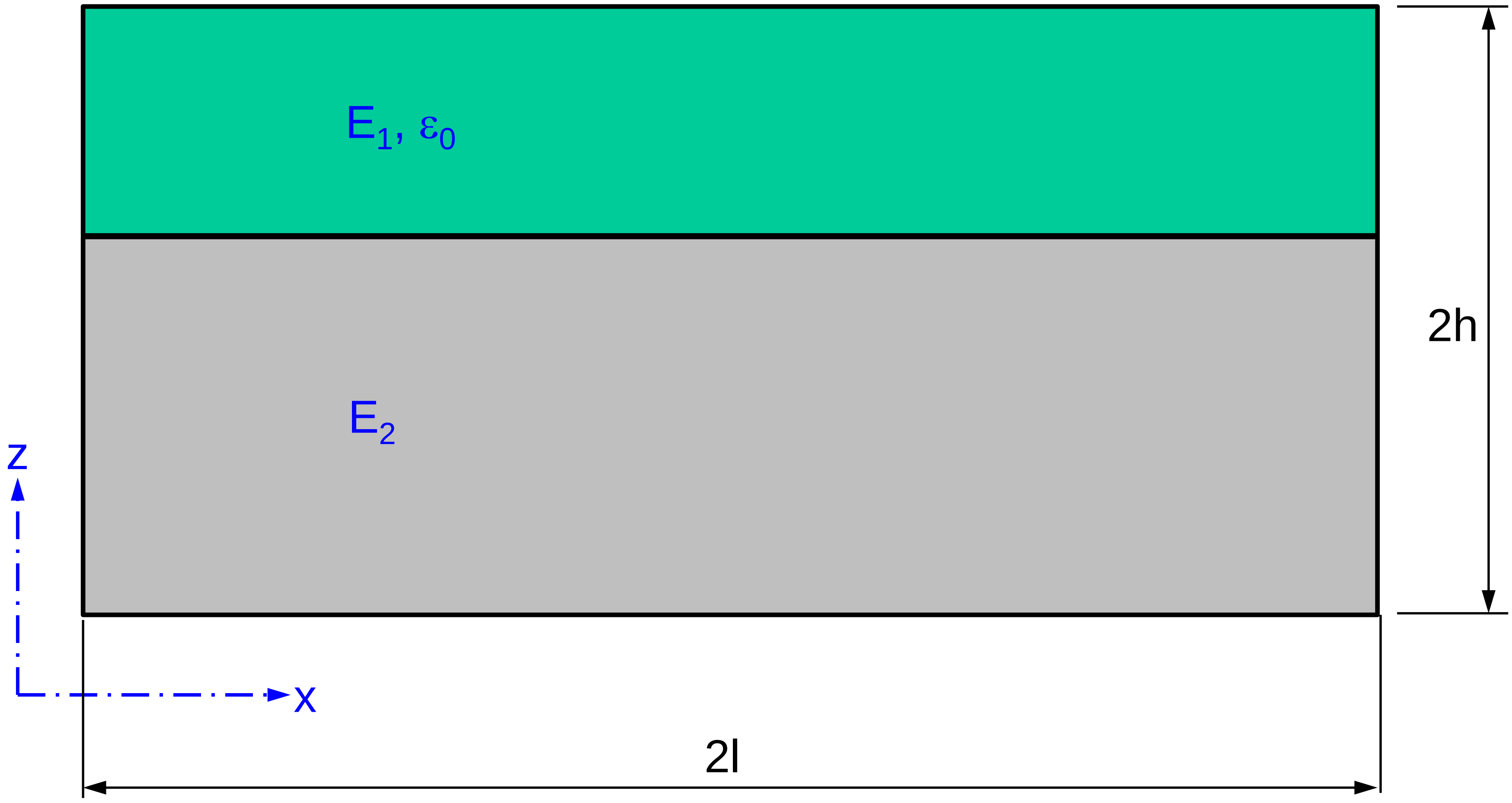
Motivation

Layered structures subjected to eigenstrains appear in many engineering applications.

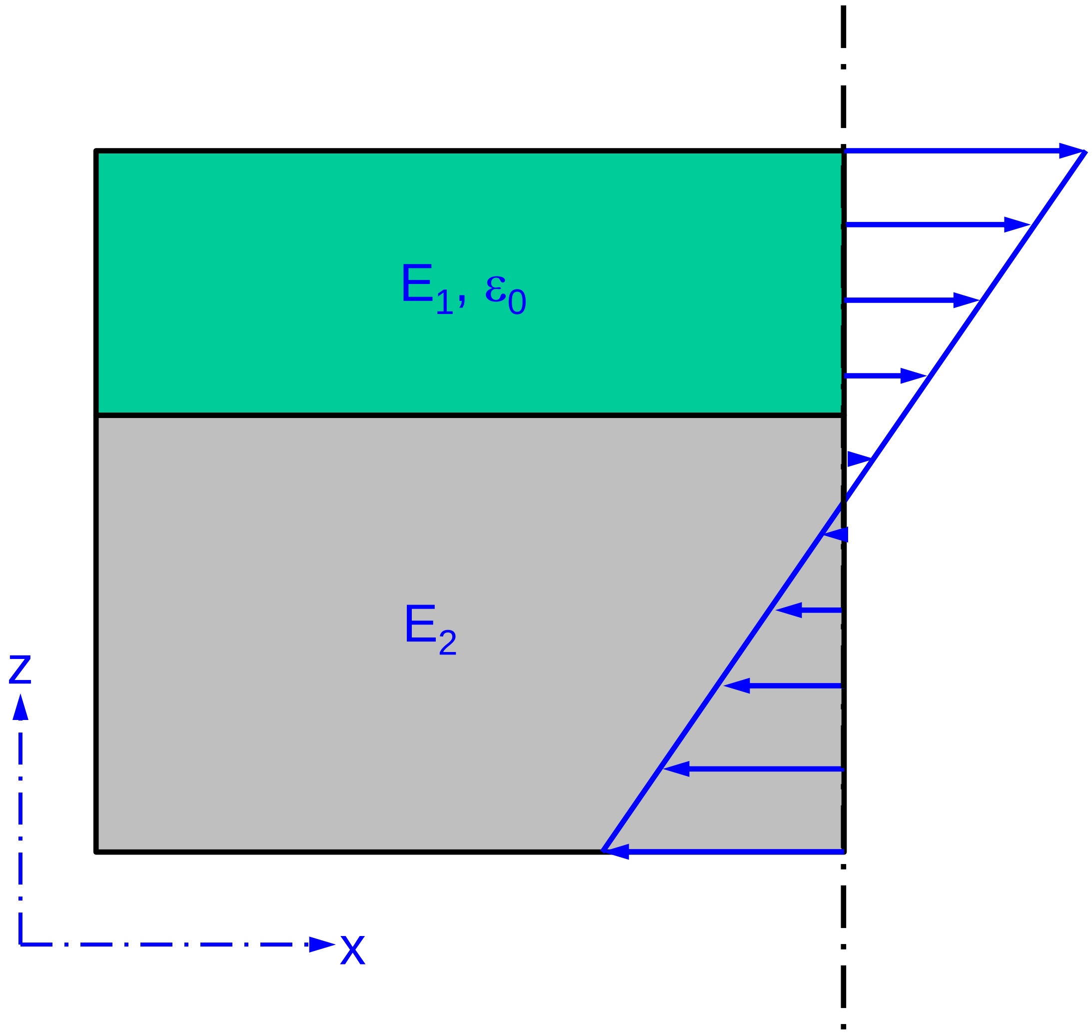
- Coated materials: Typically the coating process creates residual stresses
- Thermally loaded composites, when the thermal expansion coefficients are different
- Electronic devices are typically a layered setup subjected to thermal loads.
- Actuators sometimes take advantage of the different properties of bi-metals. Even nature builds actuators according to that principle!
- etc.

In many instances delamination is an issue. So determining the interface stresses is of vital interest.

Setup: Upper layer subjected to eigenstrain ε_0

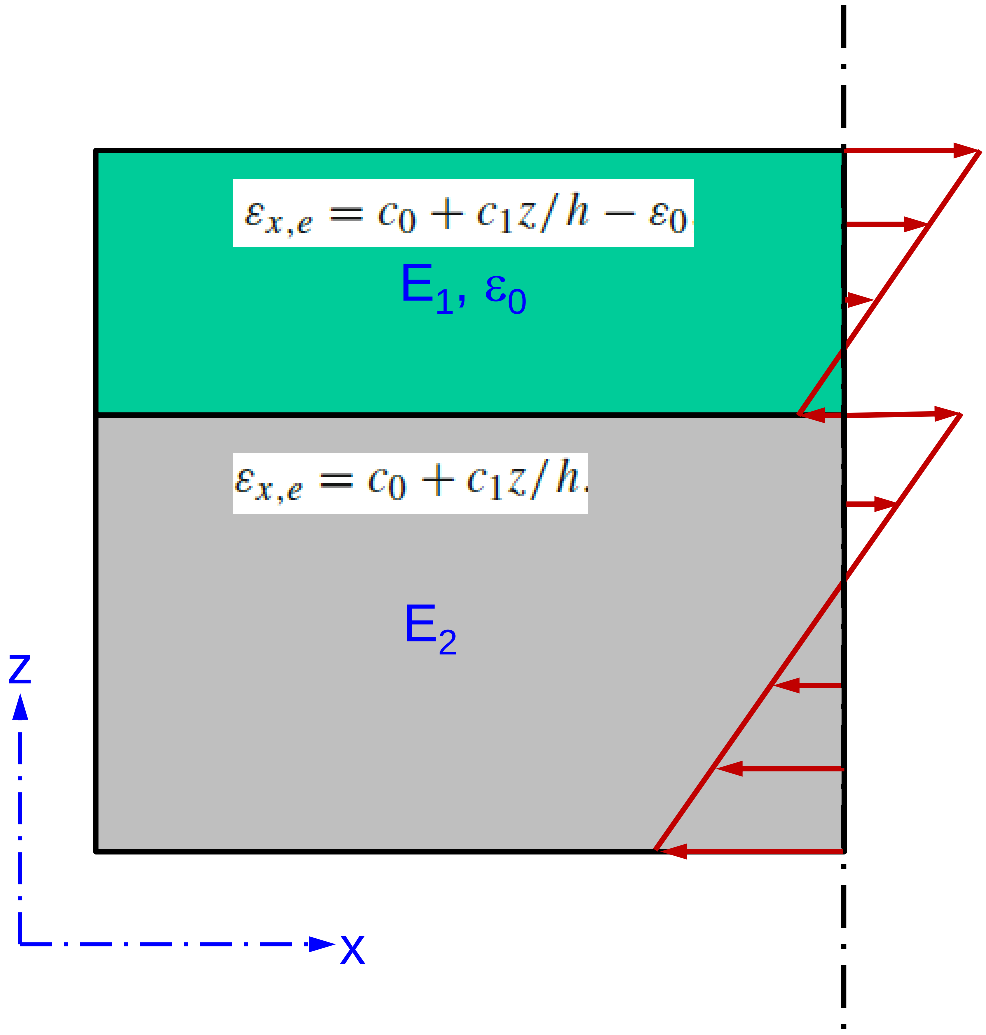


Preliminary Considerations



$$u(x, z) = (c_0 + c_1 z/h)\, x$$

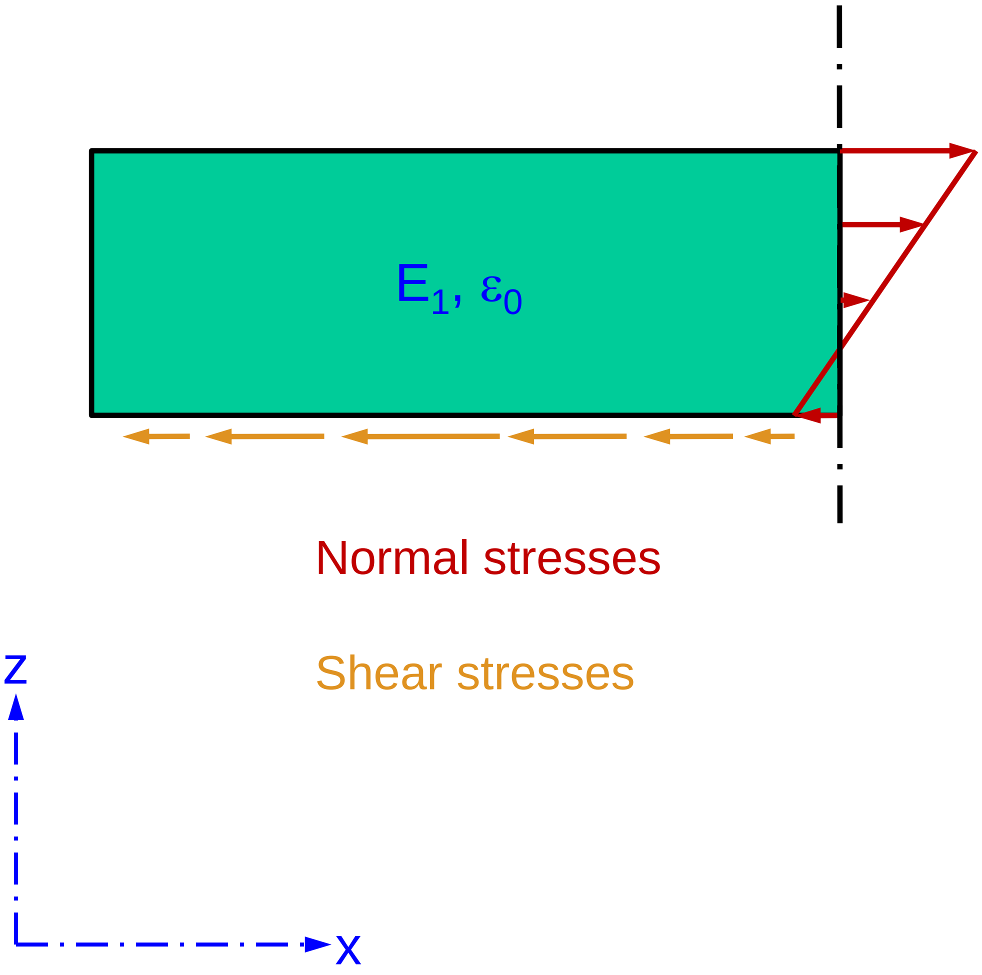
Preliminary Considerations



$$N = \int_{-h}^{h} \sigma_x \mathrm{d}z = 0$$ ✓

$$M = \int_{-h}^{h} \sigma_x \cdot z \, \mathrm{d}z = 0$$ ✓

Preliminary Considerations



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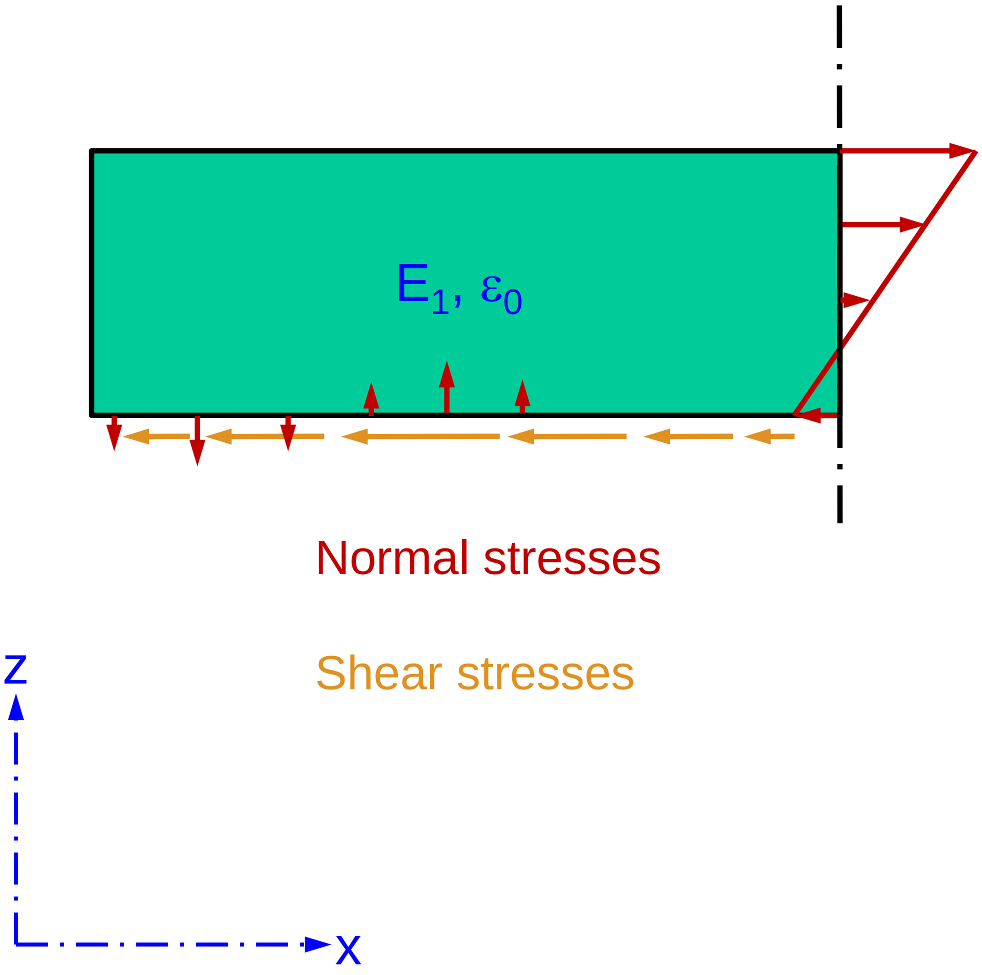
Preliminary Considerations



$$N = \int_{-h}^{h} \sigma_x \mathrm{d}z = 0$$

$$M = \int_{-h}^{h} \sigma_x \cdot z \, \mathrm{d}z = 0$$

Analytical Approach

Airy stress function, for the solution of a 2D elasto-static problem:

Stresses have to satisfy
Equilibrium conditions

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} = 0$$

Strains have to satisfy
Compatibility conditions

$$\frac{\partial^2 \varepsilon_z}{\partial x^2} + \frac{\partial^2 \varepsilon_x}{\partial z^2} = \frac{\partial^2 \gamma_{zx}}{\partial z \, \partial x}$$

The material obeys
Hooke's law

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_z) + \alpha T$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_x) + \alpha T$$

$$\gamma_{zx} = \frac{1}{G}\tau_{zx}$$

E … Young's modulus

G … Shear modulus

α … CTE

Analytical Approach

These equations are combined into a scalar function $\phi(x,z)$ which satisfies:

$$\Delta\Delta\phi = 0 \qquad \text{with } \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$$

subject to the boundary conditions of the given problem.

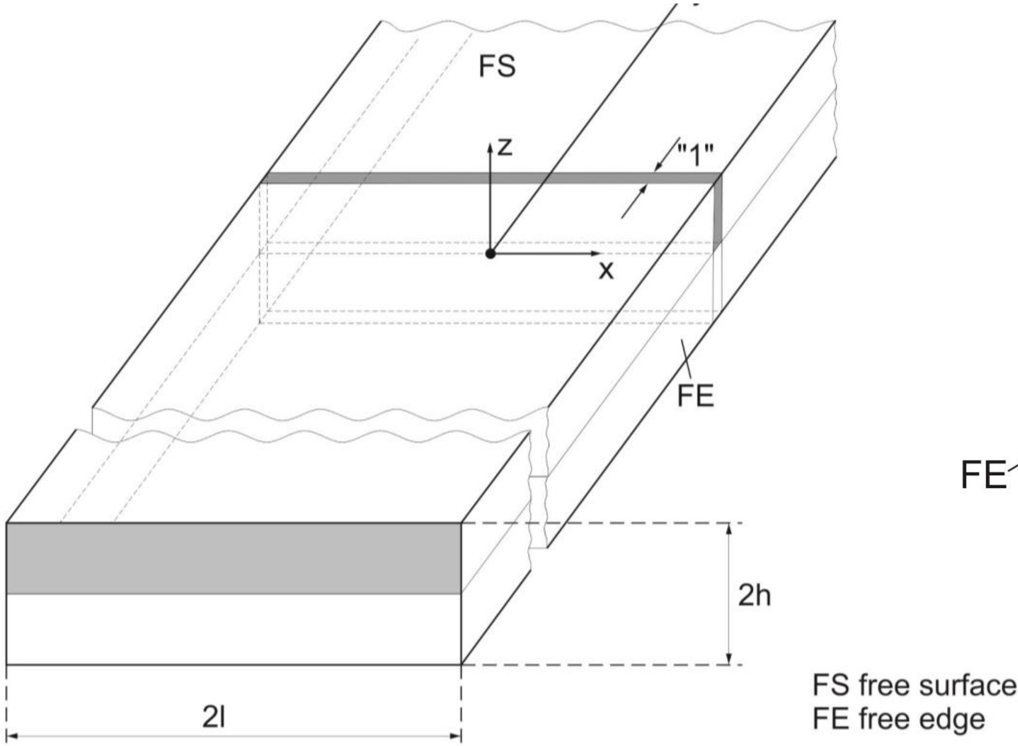
Once $\phi(x,z)$ has been determined the stress field can be calculated according to

$$\sigma_x = \frac{\partial^2\phi}{\partial z^2}, \qquad \sigma_z = \frac{\partial^2\phi}{\partial x^2}, \qquad \tau_{xy} = -\frac{\partial^2\phi}{\partial x \partial z}$$

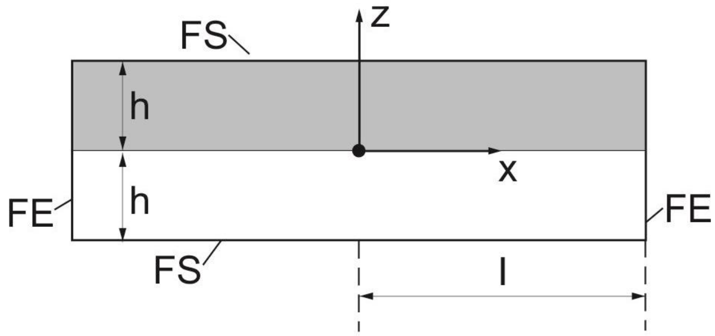
The literature offers the solutions for $\phi(x,z)$ for a variety of fundamental problems of elasto-statics.

Analytical Approach

The case to be investigated is a bilayered composite with an eigenstrain in the upper layer.

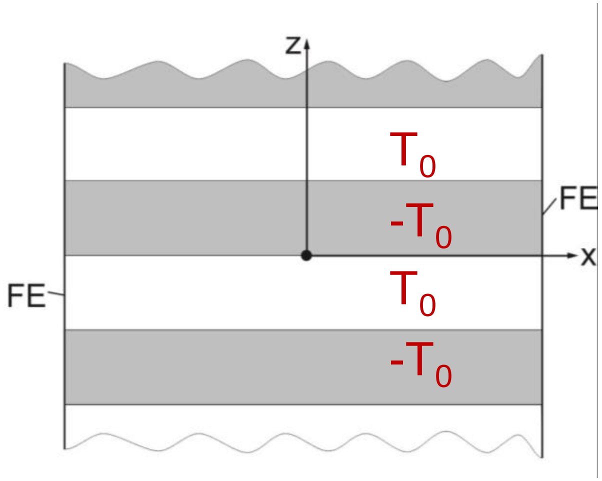


2D „Free Boundary“ (FB-model)



Analytical Approach

There is no solution for the Airy stress function $\phi(x,z)$ for the FB model. However, there is a solution to the related problem of a periodic sequence of layers with alternating eigenstrains (PC-model):



The eigenstrain field is mimicked by a temperature distribution:

$$T(z) = \sum_{j=0}^{\infty} T_0 \cdot \frac{4l}{h} \cdot \frac{1}{\beta_j l} \sin \beta_j z, \quad \beta_j = \frac{(2j+1)\pi}{h}$$

Analytical Approach

Then the stress function $\phi(x, z)$ reads:

$$\phi = \sum_{j=0}^{\infty} \phi_j (x, z)$$

with

$$\phi_j = \left(A_j \mathrm{ch} \beta_j x + B_j \frac{x}{l} \mathrm{sh} \beta_j x \right) \sin \beta_j z + \tilde{E} \alpha T_0 \frac{4}{h} \frac{1}{\beta_j^3} \sin \beta_j z$$

and

$\tilde{E} = E / (1 - \nu)$	For plane strain
$\tilde{E} = E$	For plane stress
$\tilde{E} = E / \left(1 - \nu^2\right)$	For generalized plane strain (approx.)

Analytical Approach

This gives the following stress field:

$$\sigma_x^j(x,z) = -\tilde{E}\alpha T_0 \cdot \frac{4l}{h} \cdot \frac{1}{\beta_j l}\left[A_j \mathrm{ch}\left(\beta_j l \cdot x/l\right) + B_j x/l \cdot \mathrm{sh}\left(\beta_j l \cdot x/l\right) + 1\right] \sin \beta_j z$$

$$\sigma_z^j(x,z) = \tilde{E}\alpha T_0 \cdot \frac{4l}{h} \cdot \frac{1}{\beta_j l}\Big[A_j \mathrm{ch}\left(\beta_j l \cdot x/l\right) + B_j \left(2/\beta_j l \cdot \mathrm{ch}\left(\beta_j l \cdot x/l\right)\right.$$
$$\left.+ x/l \cdot \mathrm{sh}\left(\beta_j l \cdot x/l\right)\right)\Big] \sin \beta_j z$$

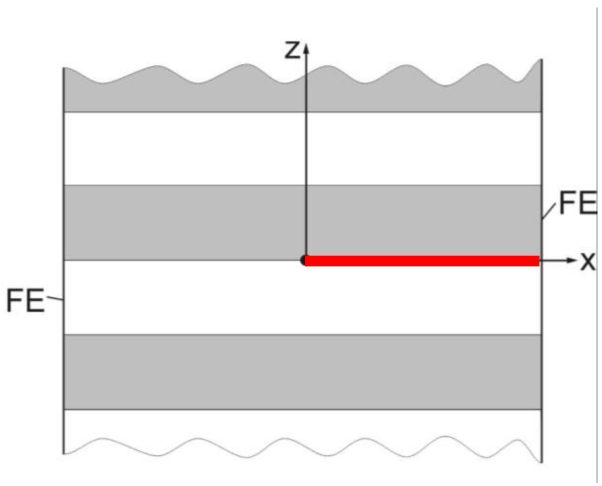
$$\tau_{xz}^j(x, z=0) = -\tilde{E}\alpha T_0 \cdot \frac{4l}{h} \cdot \frac{1}{\beta_i l}\left[A_j \mathrm{sh}\left(\beta_j l \cdot x/l\right) + \right.$$
$$\left. + B_j \left(x/l \cdot \mathrm{ch}\left(\beta_j l \cdot x/l\right) + 1/\beta_j l . \mathrm{sh}\left(\beta_j l \cdot x/l\right)\right)\right]$$

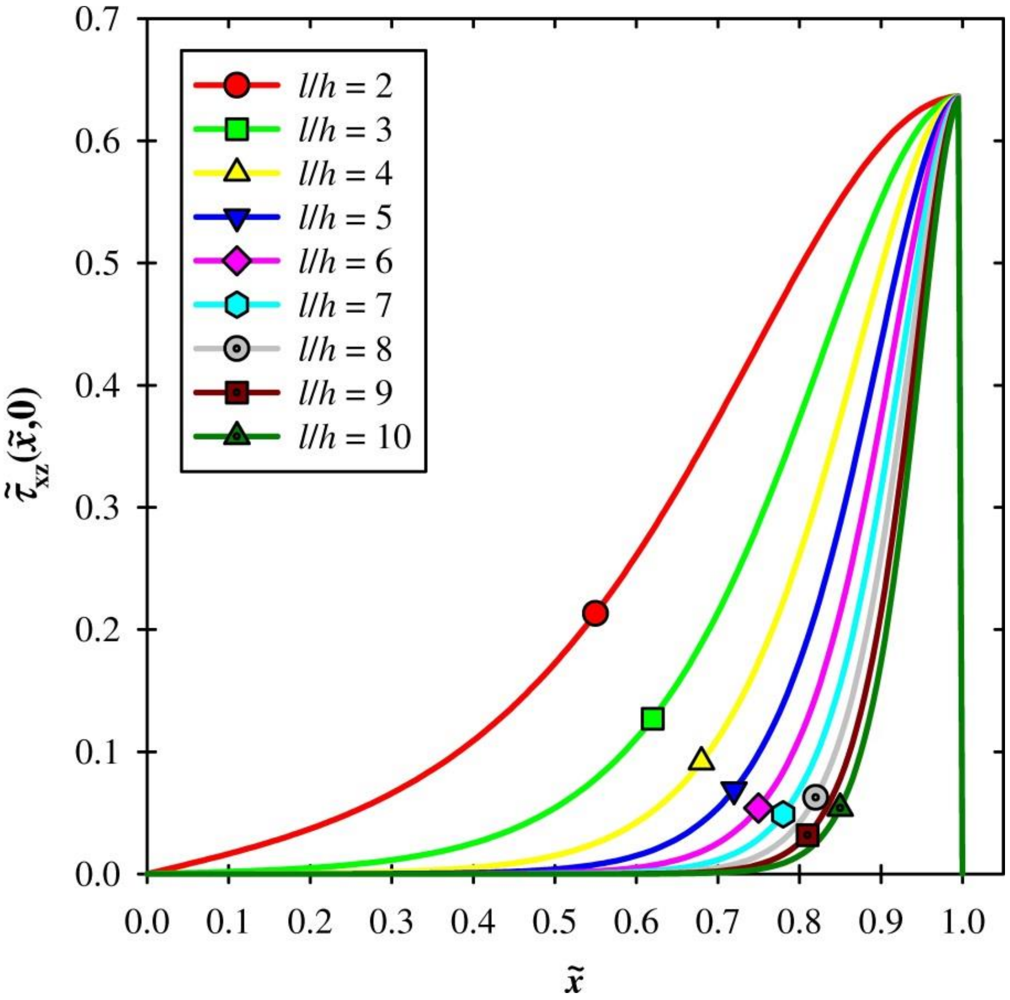
which satisfies the boundary conditions: $\sigma_x^j(x=l, z) = 0$

$\tau_{xz}^j(x=l, z) = 0$

Results of the Analytical Approach

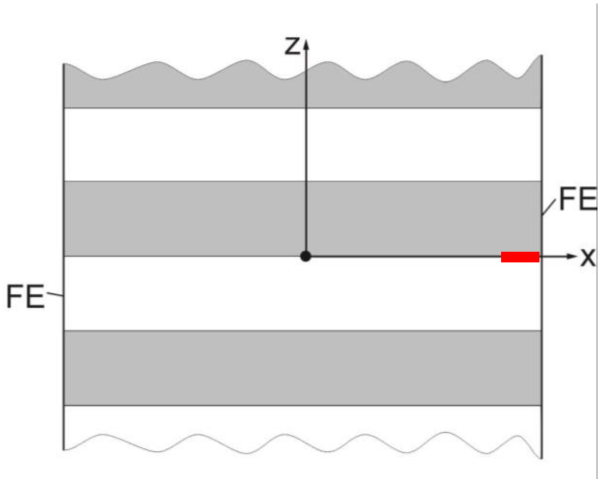
Shear stresses along x-direction:

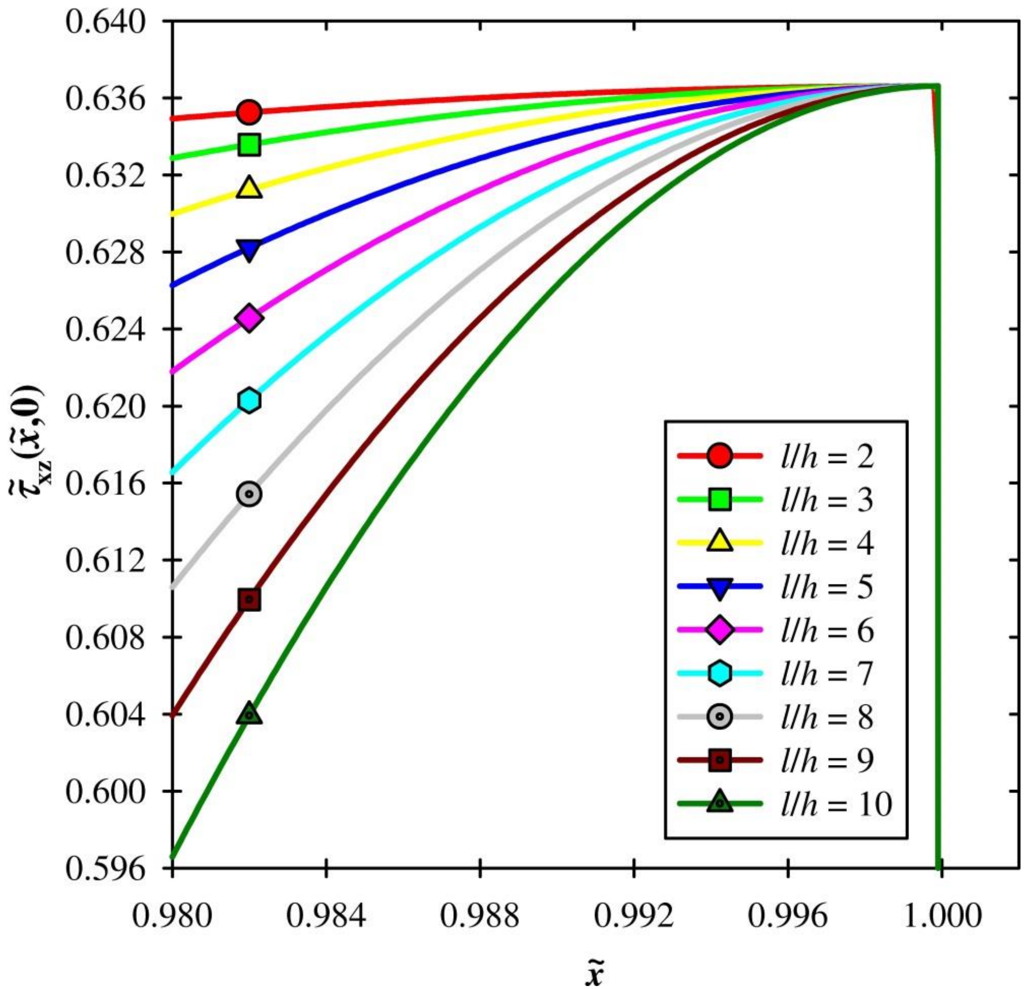




Results of the Analytical Approach

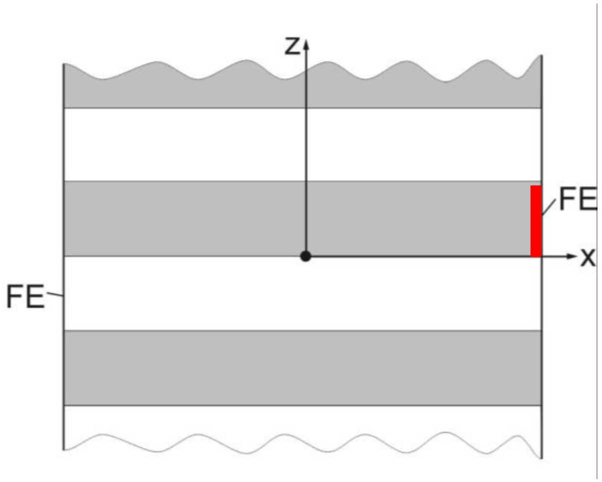
Detail at the free edges:

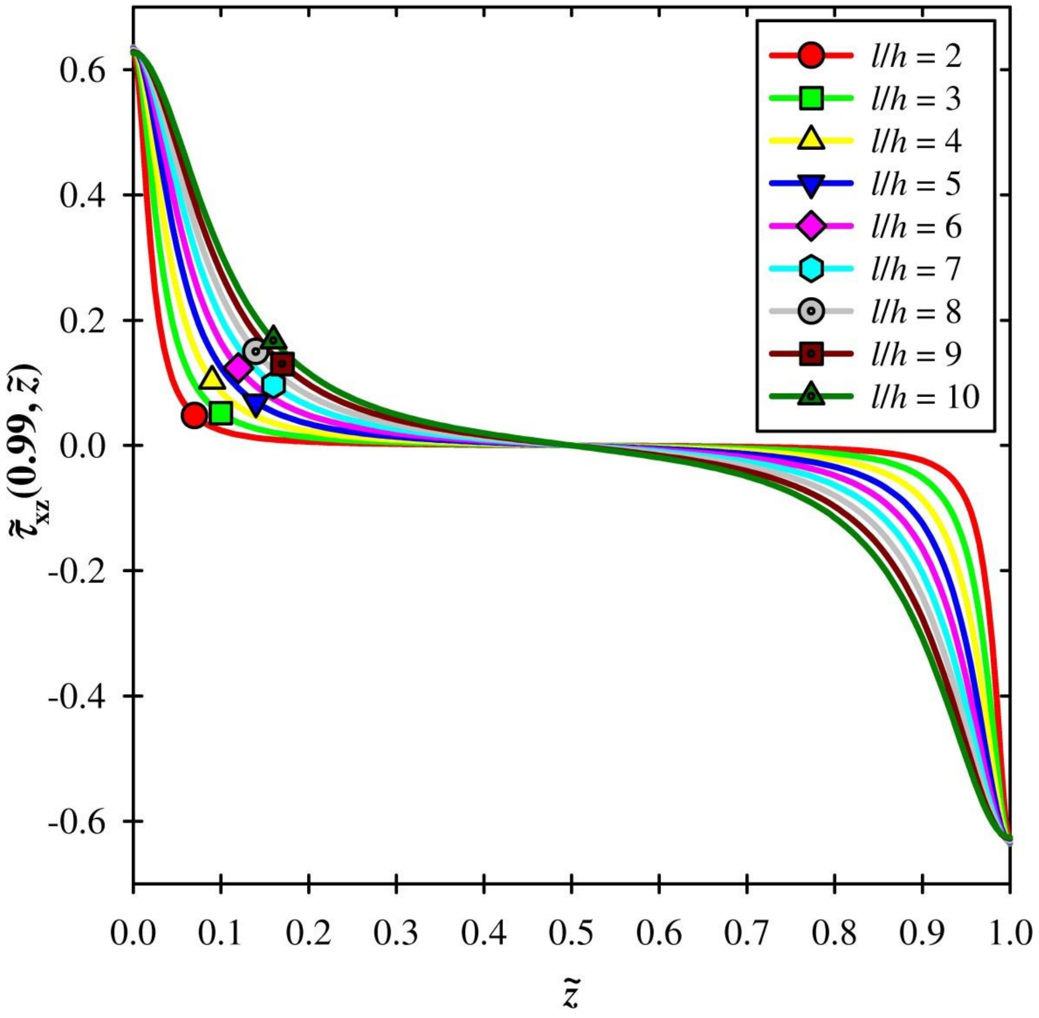




Results of the Analytical Approach

Shear stresses along z-direction, very close to free edge:

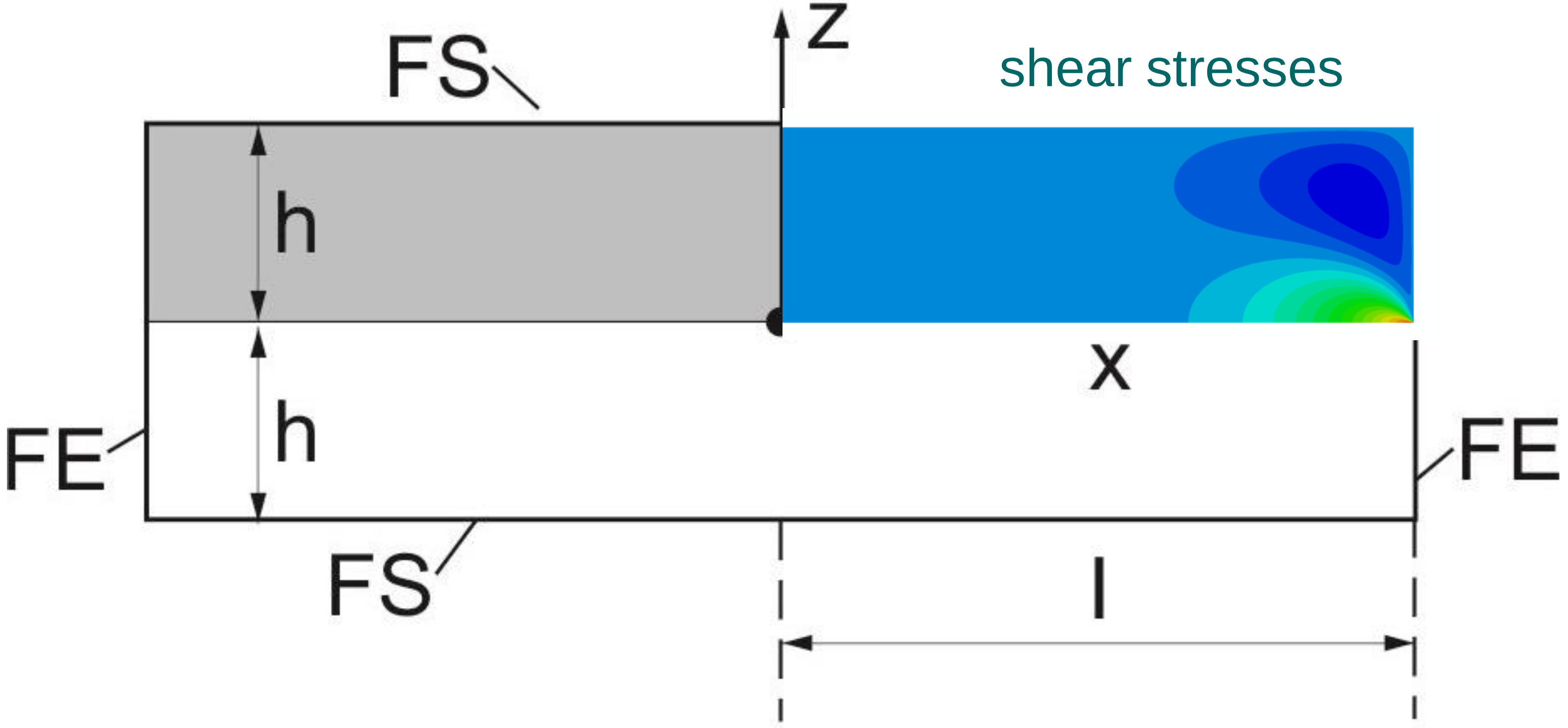




Numerical Approach

The FB model shows very similar features. A full solution can, however, not be worked out analytically, but must be computed numerically.

Strong gradients call for extremely refined meshes towards the free edges.



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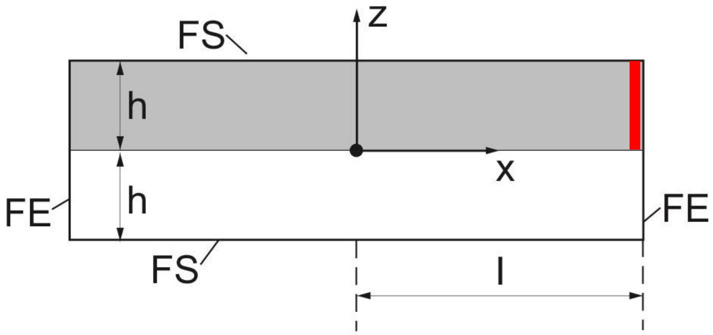
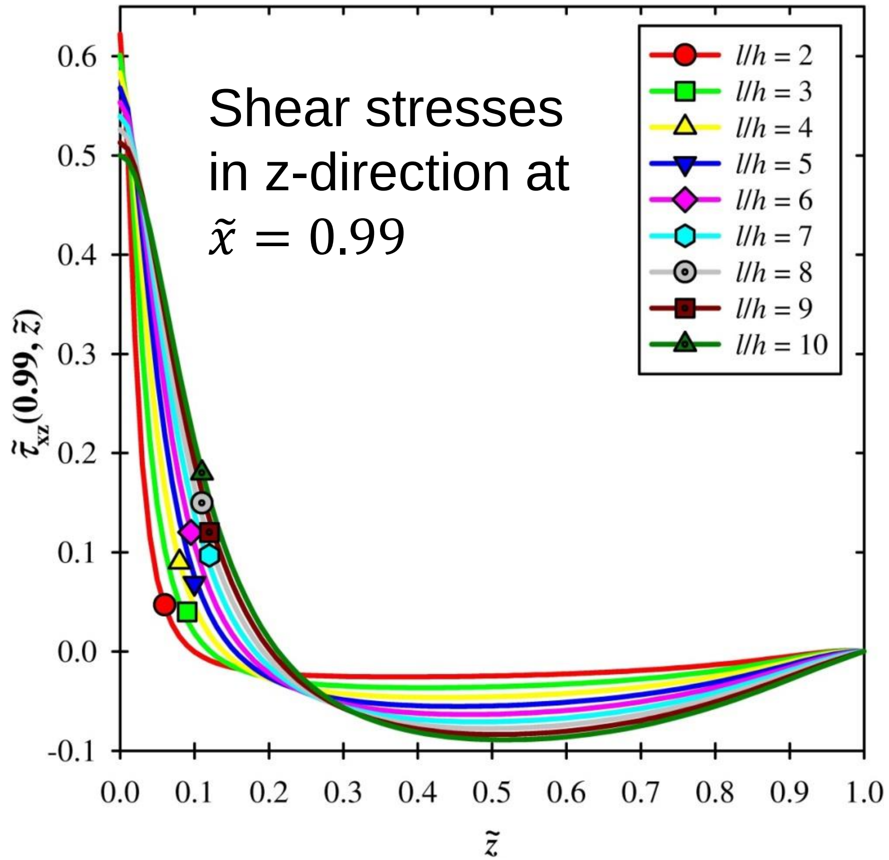
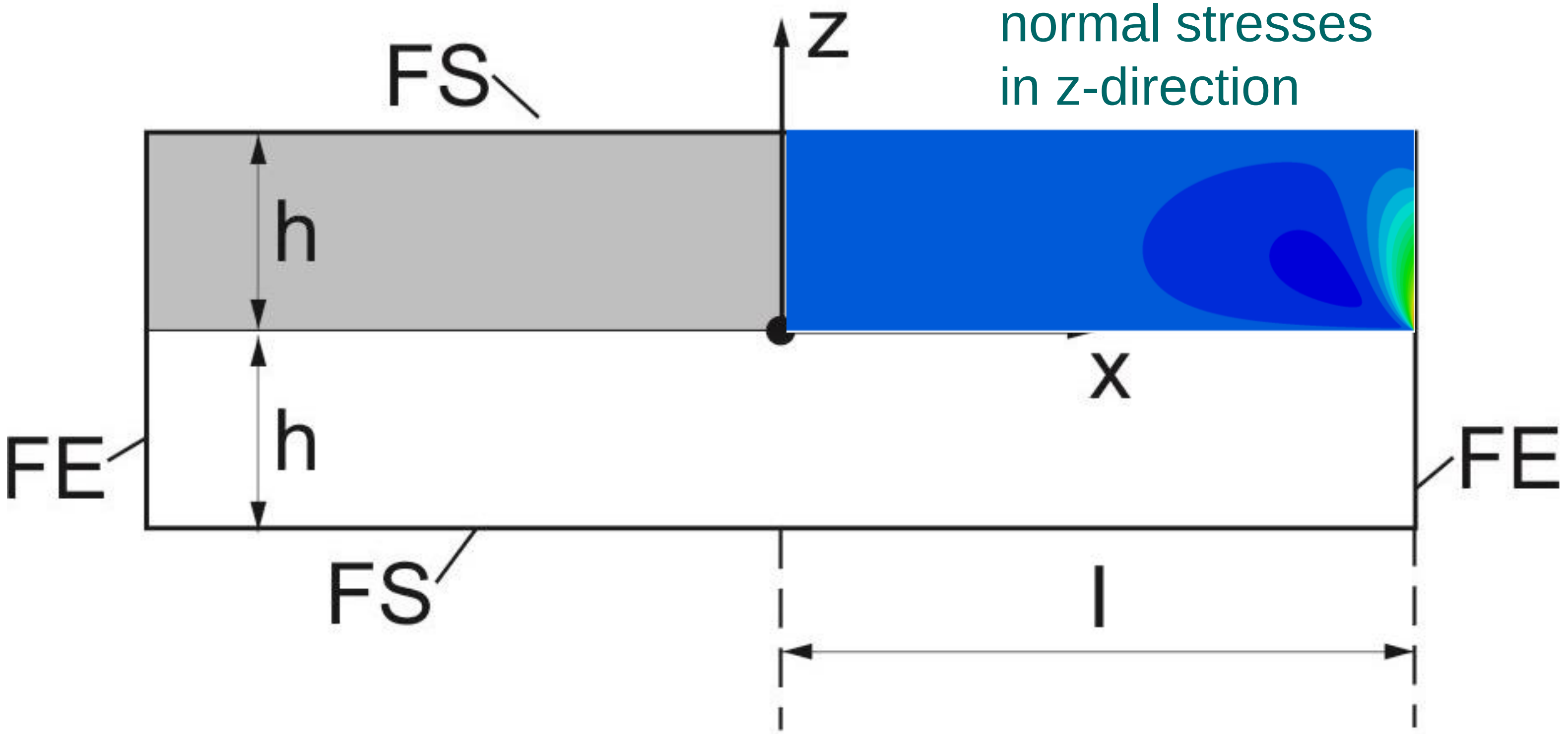


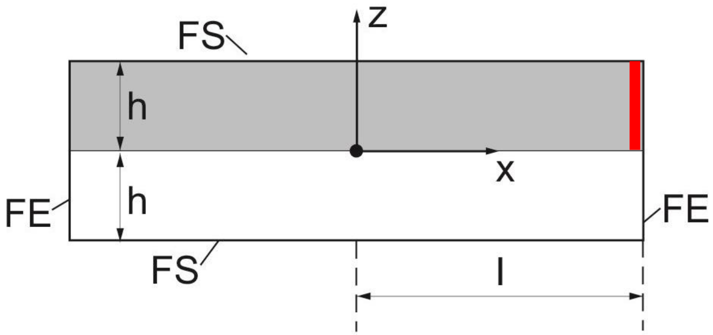


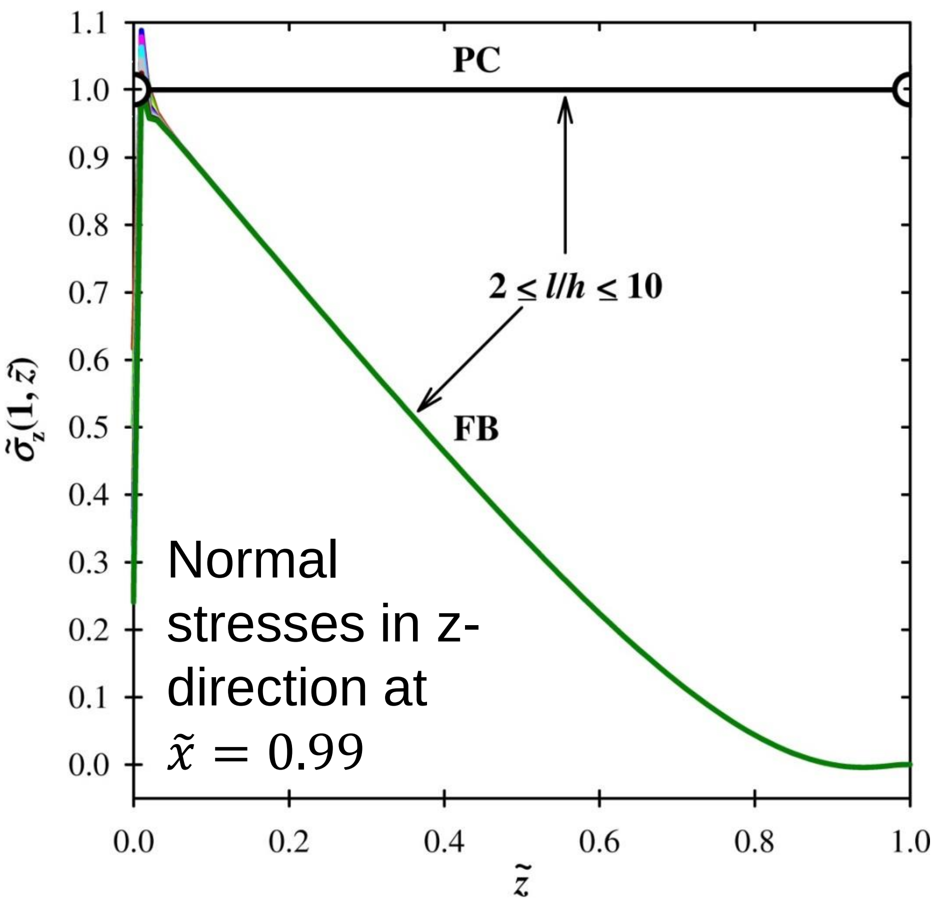
Numerical Approach



Numerical Approach

Significant differences between PC and FB arise in the normal stresses close to the free edge





Normal stresses in z-direction at $\tilde{x} = 0.99$

Conclusions

- Layered structures subjected to inhomogeneous eigenstrains appear in many engineering applications.
- For investigating delamination problems the interface stresses must be determined.
- An analytical approach has been presented for the related problem of a periodic sequence of layers.
- The solutions shows surprisingly high stress gradients at free edges. Note that these gradients have nothing to do with singularites!