





#### Neuerungen und einige Gedanken zu Materialmodellen für Kunststoffe in LS-DYNA

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## Introduction



## Introduction



### **Introduction: Summary!**



## **Constitutive Models for Polymers**

#### **Characteristic Structure of Plastics**



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## Some material laws for visco-plasticity in LS-DYNA

No.	Keyword	Formulation	Input
24,123	MAT_PIECEWISE_LINEAR_PLASTICITY	isotropic, el-pl, von Mises strain rate	LC table
81, 82	MAT_PLASTICITY_WITH_DAMAGE	isotropic, el-pl damage strain rate	LC LC table
89	MAT_PLASTICITY_POLYMER	isotropic, el-pl strain rate	LC parameter
141	MAT_STRAIN_RATE_SENSITIVE_ POLYMER	isotropic, el-pl strain rate	parameter parameter
168	MAT_POLYMER	isotropic, el-pl isochoric	parameter
193	MAT_DRUCKER_PRAGER	isotropic, el-pl strain-rate plastic compressibility	LC parameter parameter
187	MAT_SAMP-1	isotropic, el-pl strain rate damage plastic compressibility	LC table LC LC

## MAT\_SAMP-1

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## MAT\_SAMP-1: Yield surface



It is clear, that polymers exhibit a strong strain rate sensitive behavior, hence this curves need to be determined a different load velocities!

#### MAT\_SAMP-1: Yield surface input



Yield surface:  $f(p, \sigma_{vm}, \overline{\varepsilon}^{pl}) = \sigma_{vm}^2 - A_0 - A_1 p - A_2 p^2 \le 0$ 

To achieve a fast and easy calibration process, test results from tension, compression and shear tests can be fed directly into SAMP-1 via table definitions:



Therefore the yield surface coefficients  $A_{0,1,2}$  are calculated directly from stresses gained from a table lookup.

$$A_0 = 3\sigma_s^2 \qquad A_1 = 9(\sigma_s^2 \frac{\sigma_c - \sigma_t}{\sigma_c \sigma_t}) \qquad A_2 = 9(\frac{\sigma_t \sigma_c - 3\sigma_s^2}{\sigma_t \sigma_c})$$

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### MAT\_SAMP-1: Yield surface input



To achieve a fast and easy calibration process, test results from tension, compression, shear and biaxial tests can be fed directly into SAMP-1 via table definitions:



In this case the yield surface coefficients  $A_{0,1,2}$  are calculated each time step by a least squares approach fitting the yield stress values obtained by a table-lookup

MAT\_SAMP-1: Yield surface input



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## MAT\_SAMP-1: Yield surface input



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### MAT\_SAMP-1: Plastic flow definition



The plastic potential is defined as:  $g = \sqrt{q^2 + \alpha p^2} \iff \text{non-associated}$ 

Flow parameter correlates to plastic Poisson's ratio:

$$\alpha \propto v_p = \frac{9 - 2\alpha}{18 + 2\alpha} \le 0.5$$

Here  $\alpha$  is the angle between hydrostatic axis and plastic potential in  $p - \sigma_{_{V\!M}}$  space!

The plastic Poisson's ratio may be given as a curve as function of the equivalent volumetric strain.



## MAT\_SAMP-1: Tabulated strain rate (Perzyna type)

Elasto-plastic consistency condition  $f(\lambda, \dot{\lambda} = 0) = \sigma_{vm}^2 - A_0 - A_1 p - A_2 p^2 \le 0$ 

Visco-plastic constitutive law:

$$\dot{\lambda} = \frac{\langle \Phi(f) \rangle}{2\eta}$$
$$f(\lambda, \dot{\lambda}) - \Phi^{-1}(2\eta \dot{\lambda}) = 0$$



 $\Phi^{-1}(2\eta\dot{\lambda}) = f(\lambda,\dot{\lambda}) - f(\lambda,0)$ =  $A_0(\lambda,\dot{\lambda}) + A_1(\lambda,\dot{\lambda})p + A_2(\lambda,\dot{\lambda})p^2 - A_0(\lambda,0) - A_1(\lambda,0)p - A_2(\lambda,0)p^2$ 

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## MAT\_SAMP-1: Damage formulation

- Damage allows fitting of the unloading path, cyclic loading paths and load paths with strain softening. All of this are major effects for polymers.
- A damage variable (d or f) quantifies the part of the material cross section that no longer transmits forces due to cracks or pores.
- Isotropic damage is the simplest approach to this.
- Elastic damage affects material stiffness: Only deterioration of elastic parameters.
- Ductile damage affects material strength (deterioration of yield stress) or both material strength and material stiffness!



## MAT\_SAMP-1: Damage behavior

Damaging of the stresses by scalar curve definition  $d(\overline{\epsilon}^{pl}) = [0,1[$  (plastic damage)



## MAT\_SAMP-1: Failure

- Mesh: should be fine enough to capture localization arising before failure!
- Consideration of deformation history: stress/strain path during manufacturing may influence failure during crash event.
- Physical material data are needed up to failure, true hardening data beyond necking strain can only be obtained through reverse engineering
- Regularization: each softening problem is inherently mesh dependent, no material and/or failure law is meaningful unless it is regularized!
- Damage and failure law: Verification and validation process is needed to calibrate a damage and/or failure law to physical experiments.



### MAT\_SAMP-1: Failure definition

- All dependencies (strain rate and state of stress) are tabulated
- In a mathematical sense, a tabulated curve is the most general form of a continuous function in the Euclidian manifold – so why settle for less?
- The SAMP failure criterion can be considered a tabulated generalization of the Johnson-Cook criterion



## MAT\_SAMP-1: Damage and fading definition

- Failure onset defined by the parameter  $\Delta \overline{\mathcal{E}}_{rupt}^{\ p}$
- Further fading of the element defined by  $d_c$
- Strain rate dependent failure by an optional input curve

$$d_c = d_c(\dot{\varepsilon}^p)$$

Regularization by an input optional curve

$$\zeta = \zeta(l_c)$$

Triaxiality dependent failure by an optional input curve

$$\xi = \xi \left(\frac{p}{\sigma_{vm}}\right)$$

• Finally: 
$$d_c = d_c(\dot{\varepsilon}_p) \cdot \xi(\frac{p}{\sigma_{vm}}) \cdot \zeta(l_c)$$



### MAT\_SAMP-1: Regularization

- Failure onset defined by the parameter  $d_c \ \Delta \overline{\mathcal{E}}^{\,p}_{rupt}$
- Further fading of the element defined by
- Strain rate dependent failure by an optional input curve  $d_c = d_c(\dot{\varepsilon}^p)$
- Regularization by an input optional curve

$$\zeta = \zeta(l_c)$$

Triaxiality dependent failure by an optional input curve

 $\xi = \xi \left(\frac{p}{\sigma_{vm}}\right)$ 

• Finally: 
$$d_c = d_c(\dot{\varepsilon}_p) \cdot \xi\left(\frac{p}{\sigma_{vm}}\right) \cdot \zeta(l_c)$$
  
 $\frac{p}{\sigma_{vm}} = \frac{1}{3}, l_c = 5mm$ 



MAT\_SAMP-1: New features in R5!



# MAT\_POLYMER

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Polymer material model by Boyce et al.

Currently only available for solid elements

Two parallel mechanisms to describe deformations and derive stresses

Partial model A:	Neo-Hooke for elastic spring	
	Argon model for plastic part	
Partial model B:	Arruda-Boyce for network stiffness of polymers	





Deformation tensor:  $\mathbf{F} = \mathbf{F}_A = \mathbf{F}_B$  corresponding final stresses  $\boldsymbol{\sigma} = \boldsymbol{\sigma}_A + \boldsymbol{\sigma}_B$ 

Polymer material model by Boyce et al.

Partial model A:	Neo-Hooke for elastic spring
	Argon model for plastic part

Intermolecular barrier to deformations due to relative movements between molecules.

Multiplicative decomposition of elastic and plastic part of deformation tensor:  $\mathbf{F}_{A} = \mathbf{F}_{A}^{e} \cdot \mathbf{F}_{A}^{p}$ 

Velocity gradient:  $\mathbf{L}_{A} = \dot{\mathbf{F}}_{A} \cdot \mathbf{F}_{A}^{-1} = \mathbf{L}_{A}^{e} + \mathbf{L}_{A}^{p}$ 

Rate of deformation (el. & pl.):  $\mathbf{L}_{A}^{e} = \mathbf{D}_{A}^{e} + \mathbf{W}_{A}^{e} = \dot{\mathbf{F}}_{A}^{e} \cdot (\mathbf{F}_{A}^{e})^{-1}$ 

$$\mathbf{L}_{A}^{p} = \mathbf{D}_{A}^{p} + \mathbf{W}_{A}^{p} = \mathbf{F}_{A}^{e} \cdot \dot{\mathbf{F}}_{A}^{p} \cdot (\mathbf{F}_{A}^{p})^{-1} \cdot (\mathbf{F}_{A}^{e})^{-1} = \mathbf{F}_{A}^{e} \cdot \overline{\mathbf{L}}_{A}^{p} \cdot (\mathbf{F}_{A}^{e})^{-1}$$

elastic

Heo-Hooke for elastic part:  $\mathbf{\tau}_{A} = \lambda_{0} \ln J_{A}^{e} \mathbf{I} + \mu_{0} (\mathbf{B}_{A}^{e} - \mathbf{I})$  with  $\mathbf{B}_{A}^{e} = \mathbf{F}_{A}^{e} \cdot \mathbf{F}_{A}^{eT}$  (left Cauchy-Green) and  $J_{A}^{e} = \sqrt{\det \mathbf{B}_{A}^{e}} = J_{A}$  (Jacobian)



#### Polymer material model by Boyce et al.

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#### plastic

Flow rule 
$$\mathbf{L}_{A}^{p} = \dot{\gamma}_{A}^{p} \mathbf{N}_{A}$$
 with  $\mathbf{N}_{A} = \frac{1}{\sqrt{2} \tau_{A}} \boldsymbol{\tau}_{A}^{dev}, \quad \boldsymbol{\tau}_{A} = \sqrt{\frac{1}{2} \operatorname{tr} \left( \boldsymbol{\tau}_{A}^{dev} \right)^{2}}$   
and  $\dot{\gamma}_{A}^{p} = \dot{\gamma}_{0A} \exp \left[ -\frac{\Delta G \left( 1 - \boldsymbol{\tau}_{A} / s \right)}{k \theta} \right]$  Plastic multiplier, thermal activated where shear resistance  $\boldsymbol{s}$  is dependent on stress triaxiality.



Argon  $\sigma_{A}$  Neo-Hooke

Polymer material model by Boyce et al.  
Partial model B: Arruda-Boyce for network stiffness of polymers  
Viscous part are neglected: 
$$\mathbf{F}_{B} = \mathbf{F}_{B}^{N}$$
  
Stress-stretch relation:  $\mathbf{\tau}_{B} = \frac{nk\theta}{3} \frac{\sqrt{N}}{\lambda_{N}} \mathcal{L}^{-1} \left(\frac{\overline{\lambda}_{N}}{\sqrt{N}}\right) (\mathbf{\overline{B}}_{B}^{N} - \overline{\lambda}_{N}^{2}\mathbf{I})$   
Chain density:  $n$   
Number of rigid links:  $N$   
with  $\mathbf{\overline{B}}_{B}^{N} = \mathbf{\overline{F}}_{B}^{N} \cdot \mathbf{\overline{F}}_{B}^{NT}$ ,  $\mathbf{\overline{F}}_{B}^{N} = J_{B}^{-1/3} \mathbf{F}_{B}^{N}$ ,  $J_{B} = \det \mathbf{F}_{B}^{N}$ ,  $\overline{\lambda}_{N} = \left[\frac{1}{3} \operatorname{tr} \mathbf{\overline{B}}_{B}^{N}\right]^{\frac{1}{2}}$   
Rate of molecular relaxation:  $\mathbf{L}_{B}^{F} = \dot{\gamma}_{B}^{F} \mathbf{N}_{B}$   $\mathbf{N}_{B} = \frac{1}{\sqrt{2}\tau_{B}} \mathbf{\tau}_{B}^{dev}$ ,  $\tau_{B} = \sqrt{\frac{1}{2}} \mathbf{\tau}_{B}^{dev} : \mathbf{\tau}_{B}^{dev}$ 

where the rate of relaxation is: 
$$\dot{\gamma}_{B}^{F} = C \left( \frac{1}{\overline{\lambda}_{F} - 1} \right) \tau_{B}$$
  
with  $\overline{\lambda}_{F} = \left[ \frac{1}{3} \operatorname{tr} \left( \mathbf{F}_{B}^{F} \left\{ \mathbf{F}_{B}^{F} \right\}^{T} \right) \right]$ 

 $\frac{1}{2}$ 

 $\sigma$ 

## State of the art in flow curve generation

#### Flow curve generation



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## Flow curve extrapolation: Std. equations



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#### Flow curve extrapolation: Modified approach for std. equations



## Flow curve extrapolation: Bulge test!



### Flow curve extrapolation: FE piecewise optimization



## Flow curve extrapolation: Still an open point ?!

If we assume, that one of the previous methods was successful indeed, we must recognize that the solution is always strongly mesh size dependent if the part is deformed past the point of uniform expansion!

Why is that?

## Some thoughts about localization

### Localization phenomena

Starting point: Load curve (or table) definition exhibits softening behavior.

Engineering stresses vs. engineering strains show non-positive gradient (green data). But: True stress vs. true strain gradient is always positive (red data)!



#### Localization phenomena

Starting point: Load curve (or table) definition exhibits softening behavior.

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#### Why does strain-softening lead to localization?

Example: tension rod





$$u_{l} = \int_{0}^{l} \varepsilon \, dx = \int_{0}^{l} \varepsilon^{el} \, dx + \int_{l-\xi}^{l} \varepsilon^{pl} \, dx,$$
$$\varepsilon^{el} = \frac{\sigma_{*}}{E}$$
$$\varepsilon^{pl} = \frac{\sigma_{Y} - \sigma_{*}}{H}$$

Non-unique stress state when softening occurs

b)

- Failure displacement/strain depends on width of localization zone
- Vanishing plastic work for  $\xi \rightarrow 0$
- Stability of deformation process 1-2:

$$W_{ges}^{1-2} = -\frac{\sigma_Y^2 l}{2E} + \frac{\sigma_Y^2 \xi}{2H} \ge 0 \quad \Rightarrow \quad l < \frac{\xi E}{H}$$







#### **FEM: Mesh dependency and softening material**

**Example:** tension rod with A=1xL/n discretized with *n* finite elements:



**Result of experiment:** 





 Mean strain in one element:

$$\overline{\varepsilon} = \overline{\varepsilon}^{e} + \overline{\varepsilon}^{pl} = \frac{\sigma}{E} + \frac{1}{n} \left( \frac{\sigma - \overline{\sigma}_{y}}{H} \right)$$

Tangent operator:

$$E_{t,\text{tan}} = \left[\frac{d\overline{\varepsilon}}{d\sigma}\right]^{-1} = \left[\frac{1}{E} + \frac{1}{nH}\right]^{-1} = \frac{EHn}{nH+E}$$

Fracture energy:

$$G_t = \int \sigma \, du = \frac{1}{2n} \, E \overline{\varepsilon}_y \overline{\varepsilon}_u w_c \qquad w_c = crack \, width$$

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#### **FEM: Mesh dependency and softening material**

**Example**: tension rod with A=1xL/n discretized with *n* finite elements and localization in *m* elements:



## Poor man's regularisation

- $l_c$  = Characteristic element length
- $l_c^*$  = Characteristic element length when real world strains and deformations match simulation data.





## **Example: Localization of tension rod**

- A tension rod will be discretized with an increasing number of elements
- Softening plastic material is employed
- One element is imperfect (weakened)
- The load is applied via prescribed displacements
- The experiment will show the effects (pros/cons) of different possibilities for regularization



## **Example: Tension rod non-regularized**



[plastic strains at t=3ms and ETAN=-50 N/mm<sup>2</sup>]

## **Example: Tension rod visco-plastic formulation**



[plastic strains at t=3ms and ETAN=-50 N/mm<sup>2</sup>]

## **Example: Tension rod non-local formulation**



[plastic strains at t=3ms and ETAN=-50 N/mm<sup>2</sup>]



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## Softening and mesh dependency

**In general:** Enhance the continuum or material formulation by further information about size of failure zone (internal length parameter). Currently the following approaches are known:

- Use of Cosserat continuum (includes rotation degrees of freedom on material point level) instead of Bolzmann continuum.
- Making use of "time dependent" constitutive formulations, e. g. use of viscous material formulations in dynamic simulation. But still a certain tendency to localize within a small band may develop depending on the mesh-size, material properties and loading velocity.
- Formulations that take the size of the element into account to adjust the softening branch of the evolution law (fracture energy approach).
   Here m/n is a factor that takes the dimension of the problem (internal length parameter) into account!!!
- Non-local material formulations that take strain gradients of neighboring elements into account to determine the constitutive evolution within the actual element.

Here n/m is a held constant by defining a radius of influence – again a internal length parameter!

🛧 available in LS-DYNA

Additive failure model with scalar (isotropic) damage

## GISSMO

#### **Generalized Incremental Stress State dependent damage MOdel**



## GISSMO

#### **Generalized Incremental Stress State dependent damage MOdel**



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## **GISSMO: Schädigungsevolution**



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# Gurson vs. GISSMO

#### **Regularization of element size dependency**



### **GISSMO: Einfluss der Belastungsgeschichte**



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## Thank you for your attention.

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